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Vendredi 24 Juin 2005
Cours statistiques

9h30 – 10h30 et 11h00 – 12h00

The present lecture consists of the following three parts.

i) Short and comprehensive introduction to information geometry

Information geometry studies the intrinsic properties of manifolds of probability distributions (that is, statistical models).
A statistical model forms a Riemannian space, where the Fisher information matrix plays the role of Riemannian metric.
A statistical manifold has two dually coupled affine connections, so that it possesses two types of geodesics. When a manifold is dually flat, it permits an invariant divergence measure, which is the Kullback-Leibler divergence, and the generalized Pythagorean theorem holds. We explain these new differential-geometrical structures and their meanings in statistics without going into details of differential geometry.

ii) Statistical theory of population coding

Information is represented in the brain by a pattern of excitation of neurons. When neurons are arranged in a field (neural field), an excitation pattern is generated stochastically, depending on the stimulus given from the outside. We show the Fisher information in such a field, and study how accurate a neural population encodes information. Excitation of neurons are in general correlated. There exist not only pair wise correlations but higher-order correlations. Information geometry gives an orthogonal decomposition of correlations into a sum of pair-wise, triple-wise and so forth correlations.
An ensemble of neurons sometimes fire synchronously. We study what types of correlations exist in such a synchronized state by using information geometry.

iii) Dynamics of learning in stochastic multilayer perceptrons

Backpropagation is a well known learning algorithm for the multilayer neural networks called multilayer perceptron. The set of multilayer perceptrons forms a statistical manifold, called the neuromanifold.
Learning takes place in the neuromanifold, giving a trajectory of learning dynamics. Dynamics of learning depends on the information-geometrical structure of the neuromanifold. The peculiarity of a neuromanifold is that the Fisher information matrix (that is, the Riemannian metric) degenerates in some submanifolds. This type of singularity causes serious slow-down of learning, known as plateau phenomena. We analyze the dynamics of learning in a neighborhood of singularity, and propose an efficient new learning algorithm called the natural gradient method, which takes the geometrical structure into account.

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