

# Functional connectivity analysis : Application to motor skill learning in humans

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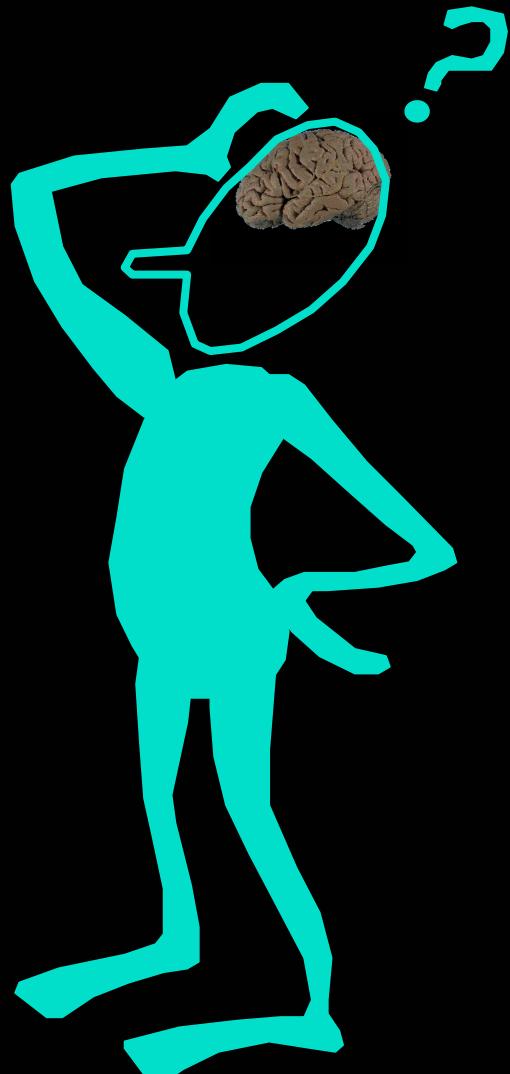
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# Functional connectivity

## Concept of brain connectivity



### Numerous definition of functional connectivity

- multiple types of data
- different spatial and temporal resolution
- functional interactivity differ from one another
- data represent neural ensemble activities
- data represent macroscopic brain regions
- different computational algorithms
- correlation within a task
- correlation between task

### Lack of definition of functional connectivity

- the relationship between the defined functional connectivity and its underlying neural substrate is unknown
- concept of functional connectivity is not well defined

# Functional connectivity : definition

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## Friston's definition

- **Functional connectivity** is defined as the temporal correlation between spatially defined brain regions
- **Effective connectivity** is defined as the influence that one brain region exerts on another.

## Other definition (Aertsen and Preissl, 1991)

**Functional connectivity** is defined as a groups of neurons that act together in a coherent fashion

# Functional connectivity : definition

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## Brain connectivity

- **Brain unit** can be thought of as the amount of brain tissue giving rise to the activity recorded in a single time series. Each unit is then associated to a *functional process*, or phenomenon, that characterizes it.

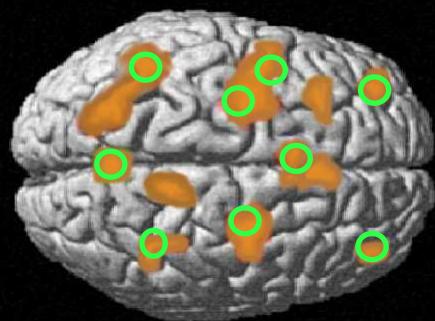
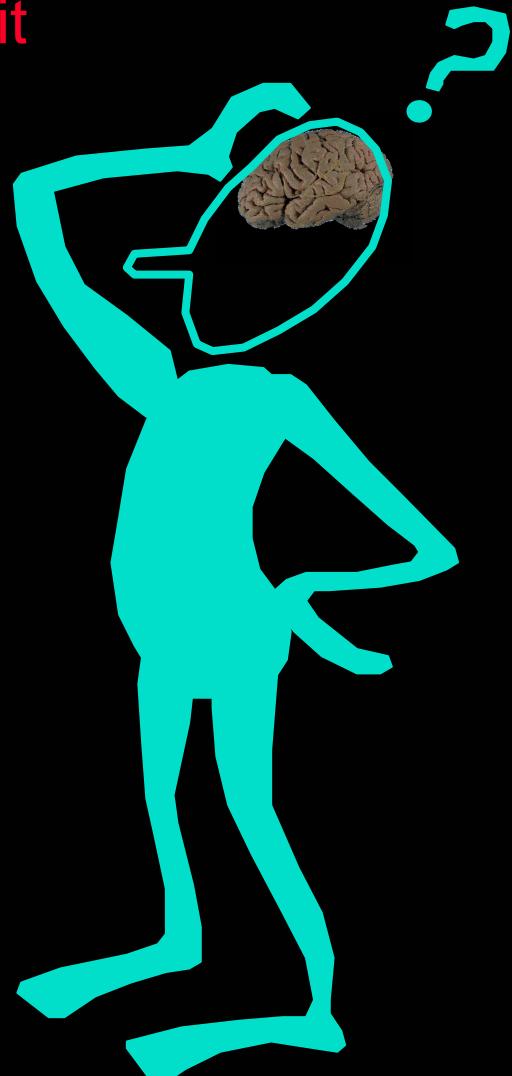
Exemple: In fMRI, they can be thought of as the BOLD contrast measured at a particular time sample.

- **Functional brain interactivity** be defined as *all potential or real information exchanges between brain units*

# Functional connectivity

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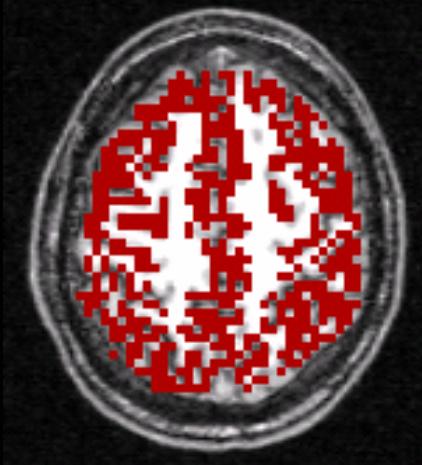
Brain unit



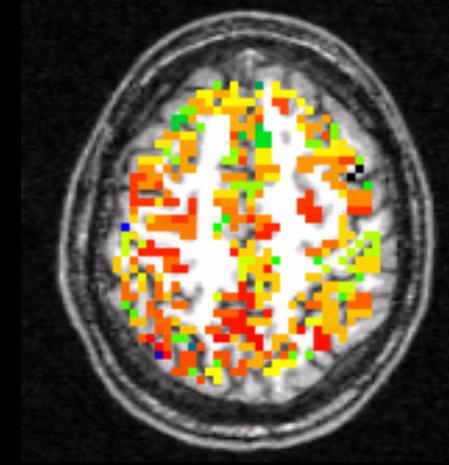
# Brain units : Competitive region growing algorithm

**Hypothesis :** Brain regions considered for functional connectivity purpose must be composed of voxels having an homogeneous temporal signal.

## Competitive region growing algorithm:



Cerebral cortex

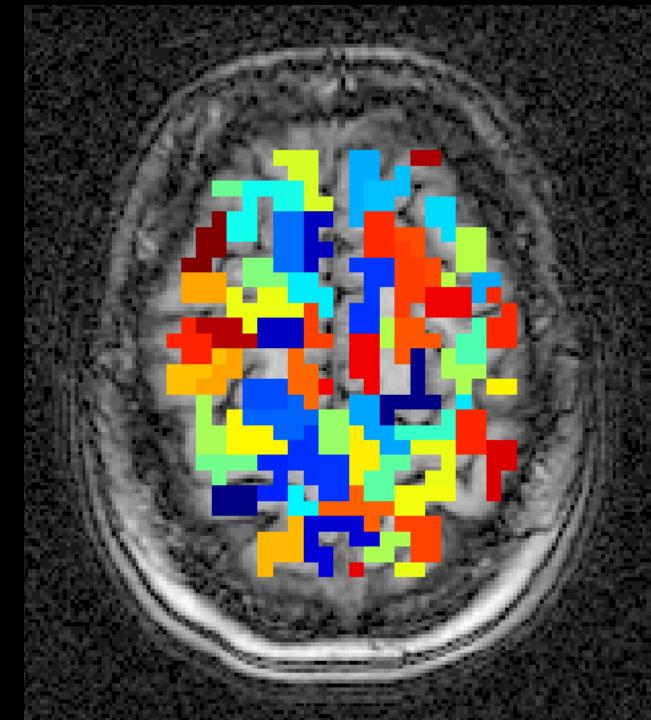
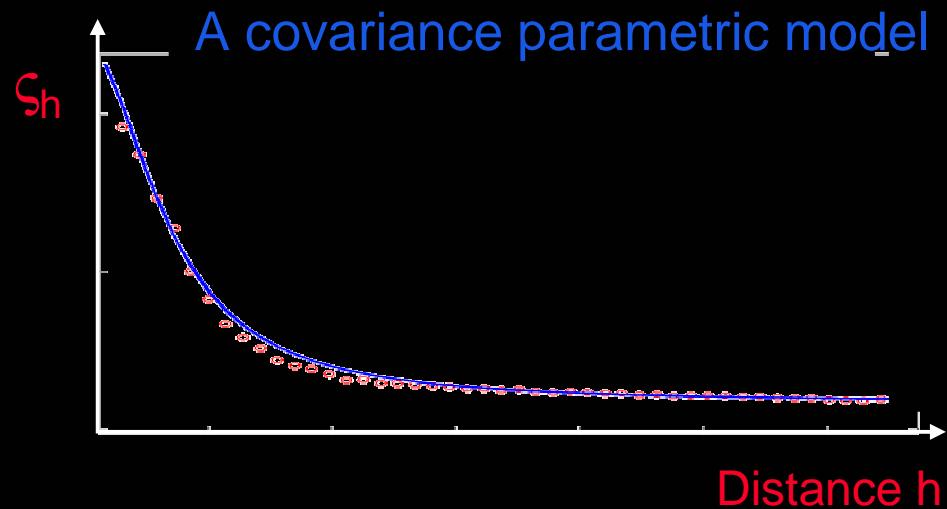


Homogeneous  
brain areas

1. Initially, clusters are all singletons.
2. Similarity measure between clusters:  
mean correlation over all pairs of voxels
3. Clusters are merged if they are MNN,  
with respect to 26-connexity, similarity  
measure, extent of the merged cluster <  
 $p$  voxels.
4. Algorithm stops when no more  
merging is possible.

# Brain unit in fMRI

**Hypothesis:** Noise in fMRI data is spatially correlated. Appropriate tests on functional connectivity must take it into account.

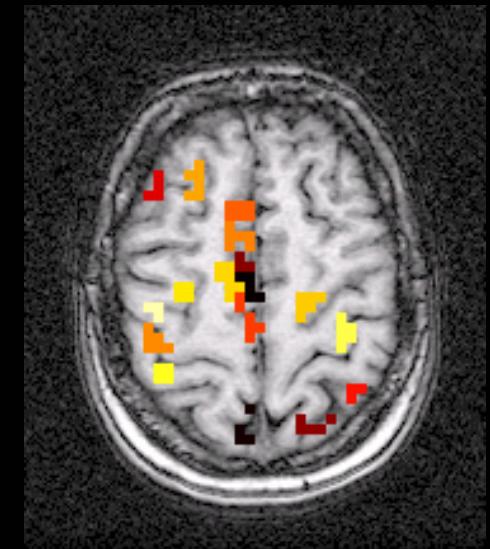
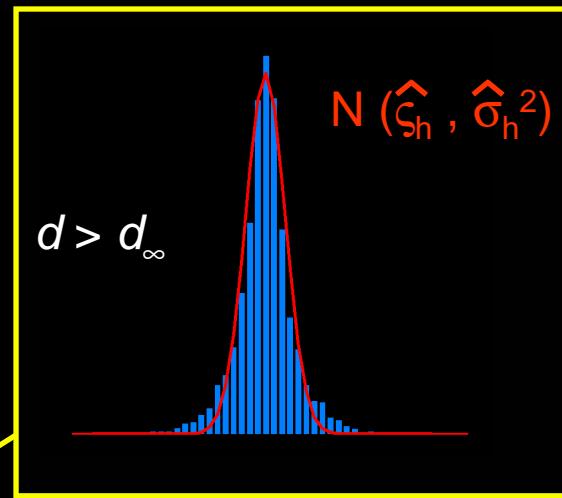
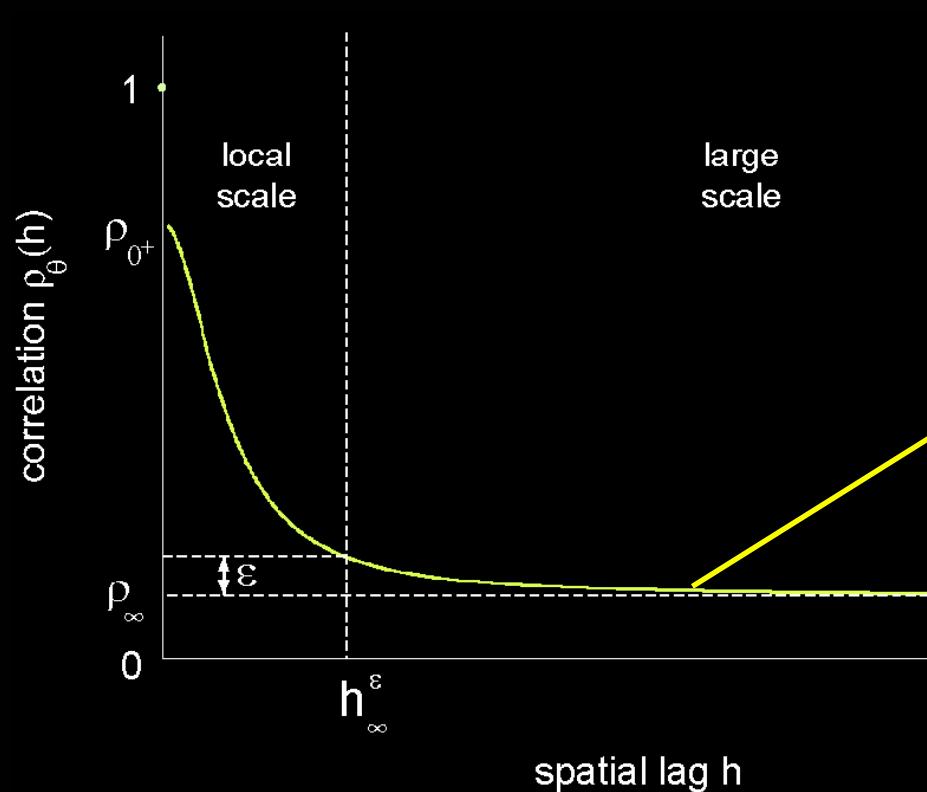


Rational-quadratic model of the covariance  
is a function of the distance between clusters

Bellec et al. (2004) /ISBI/

# Brain unit in fMRI

**Hypothesis :** As spatially close regions may correlate only due to spatial proximity, we wish to detect regions exhibiting functional connectivity with distant regions.



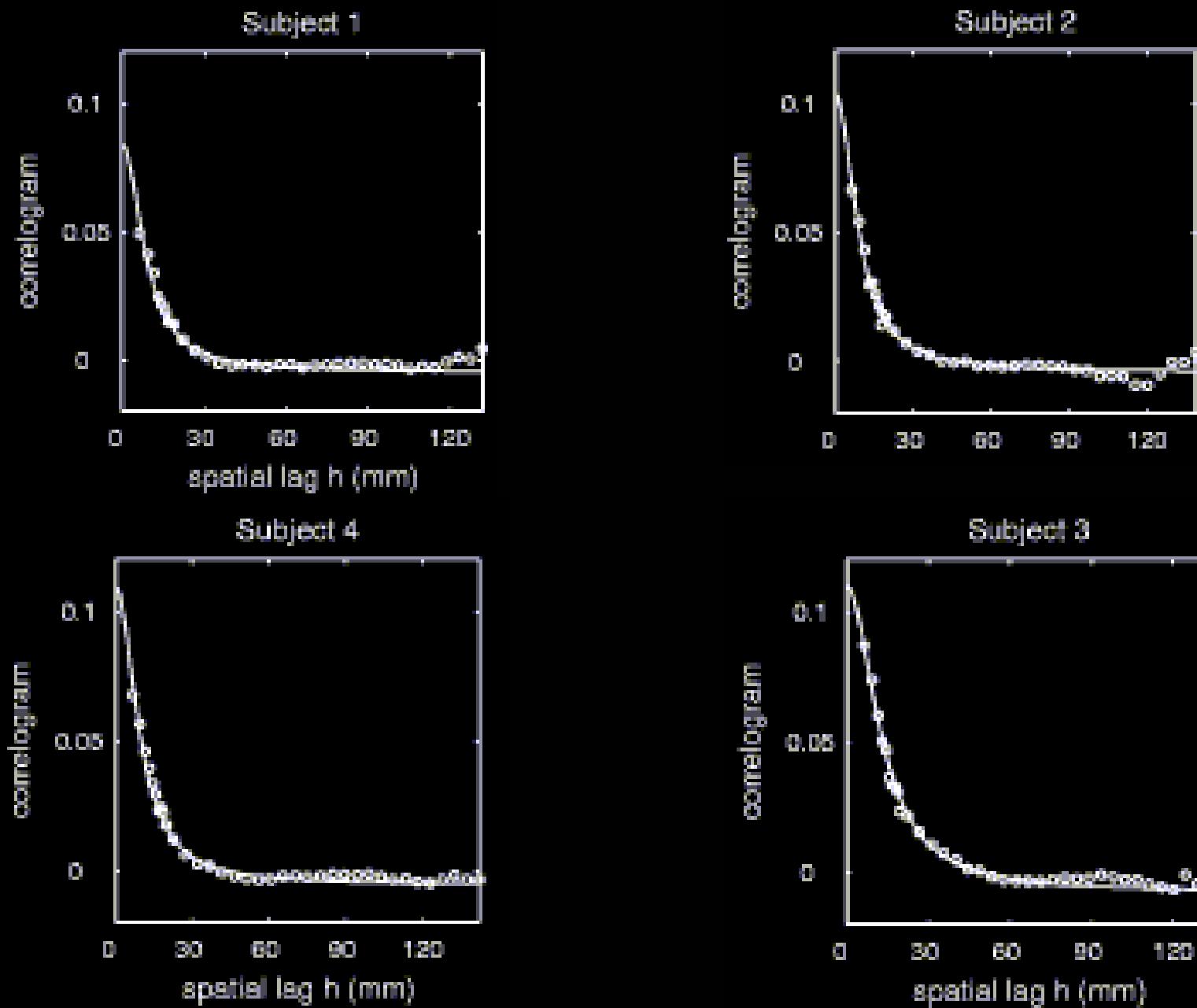
$$\sqrt{v}\mathcal{L} \sim \mathcal{N}(\mu, D)$$

$$\mu_{ij} = \rho_\infty \quad D_{ij,M} = \rho_\infty^2 + \rho_0^2$$

Identified large-scale functional brain units

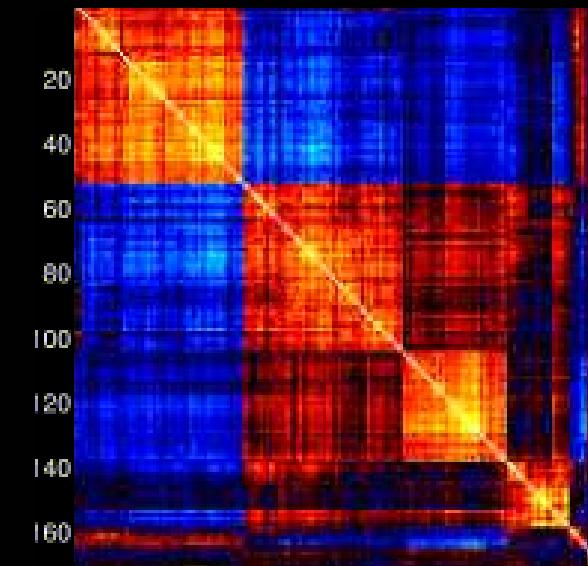
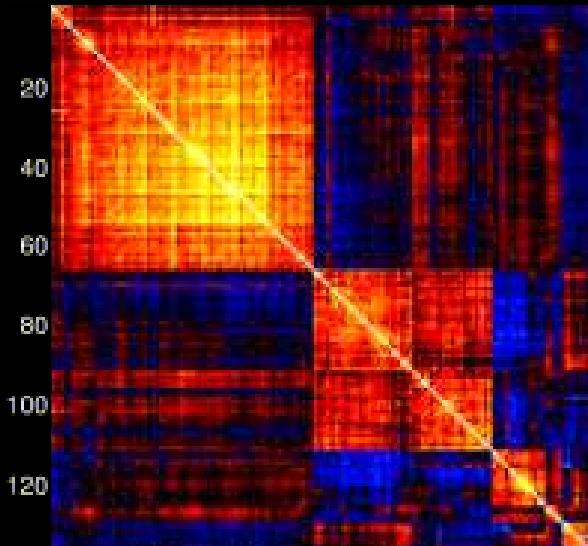
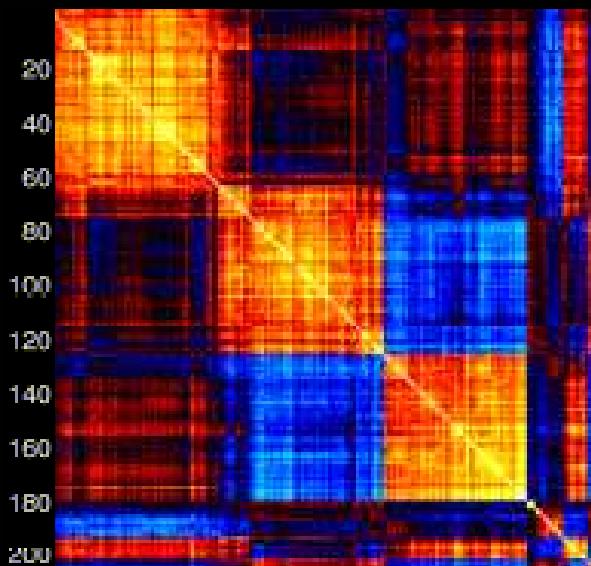
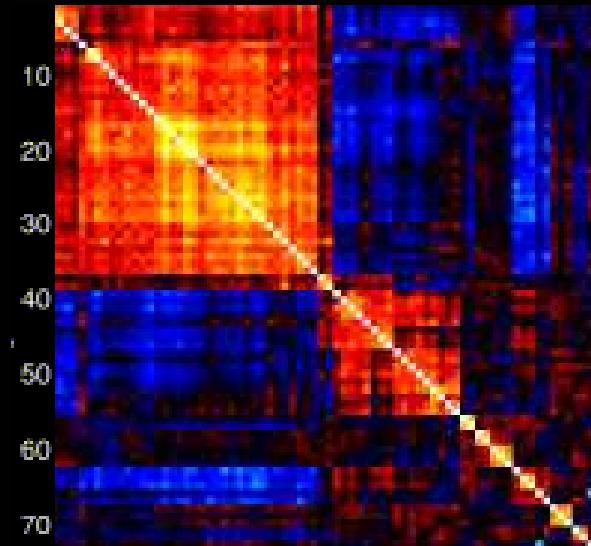
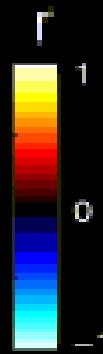
Bellec et al. *NeuroImage* (2005)

# Brain unit in fMRI

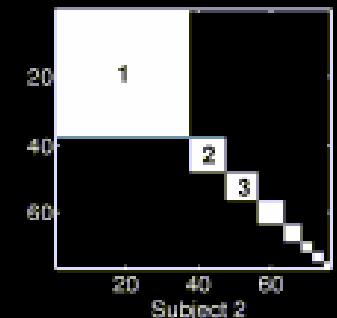


# Identification of a large-scale functional network in fMRI

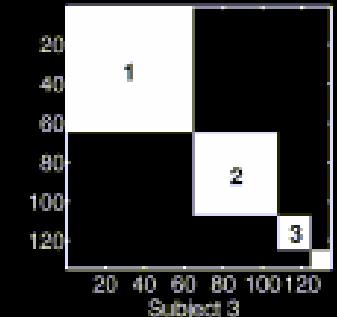
**Motor  
datasets**



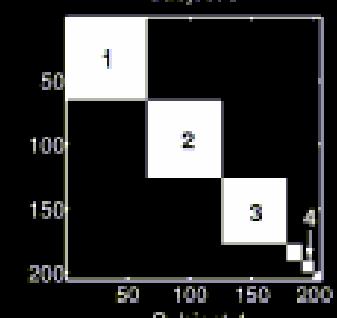
Subject 1



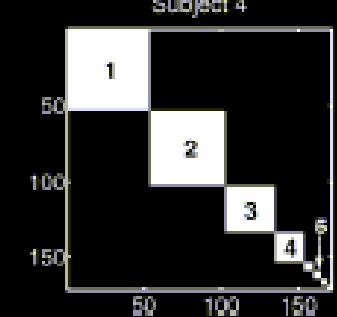
Subject 2



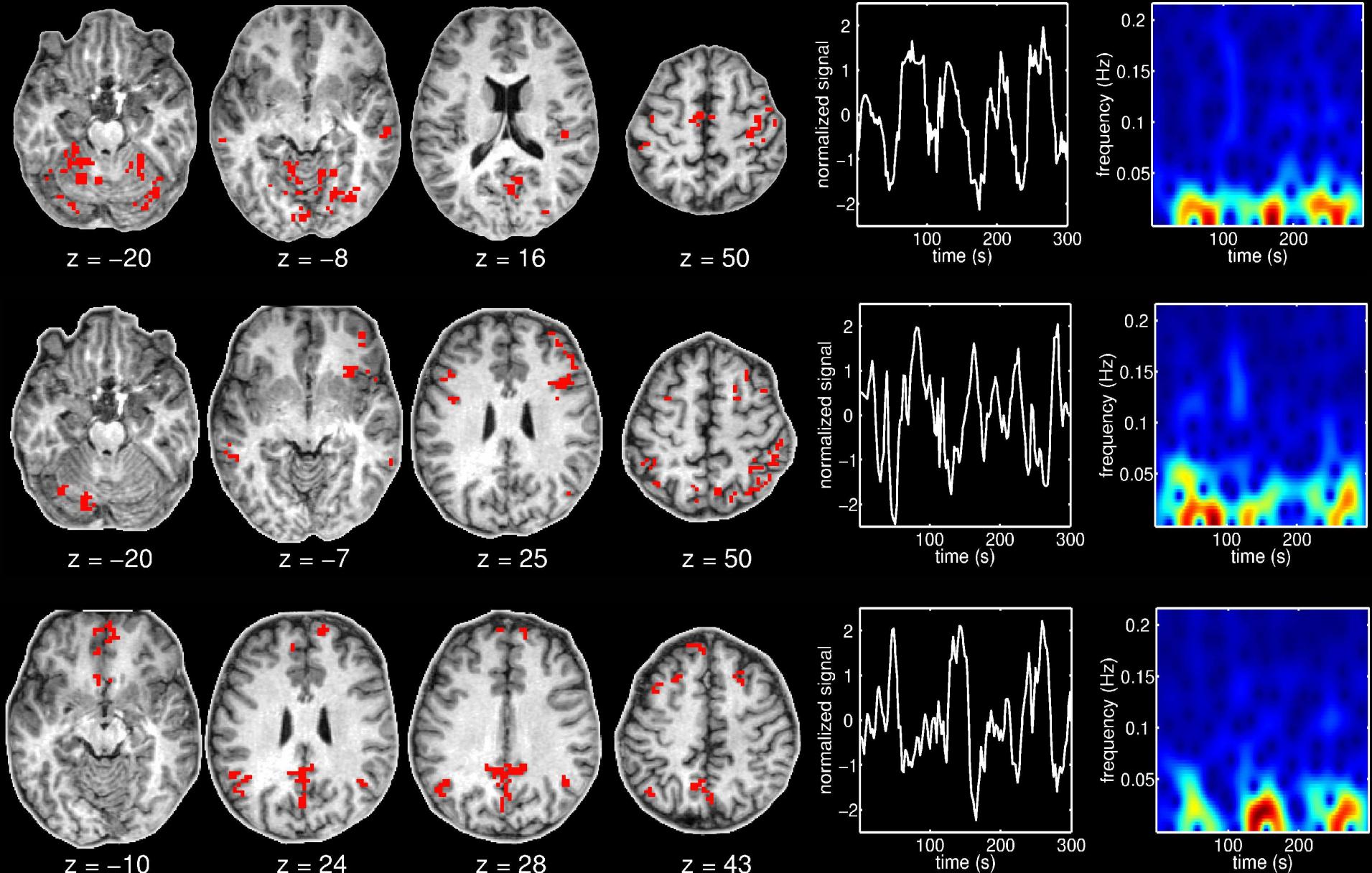
Subject 3



Subject 4



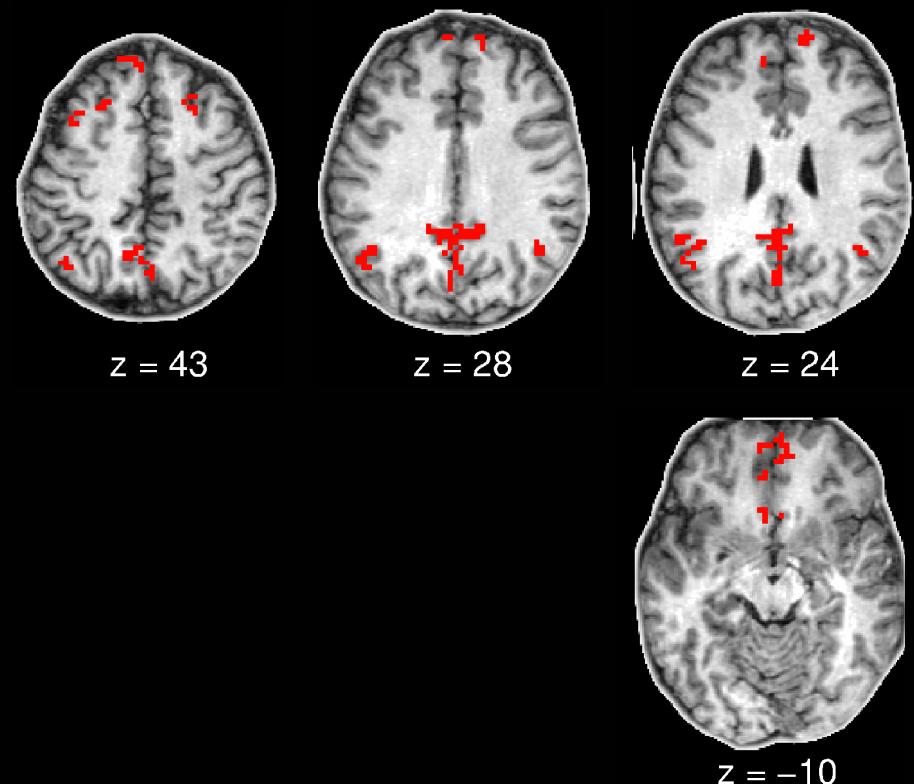
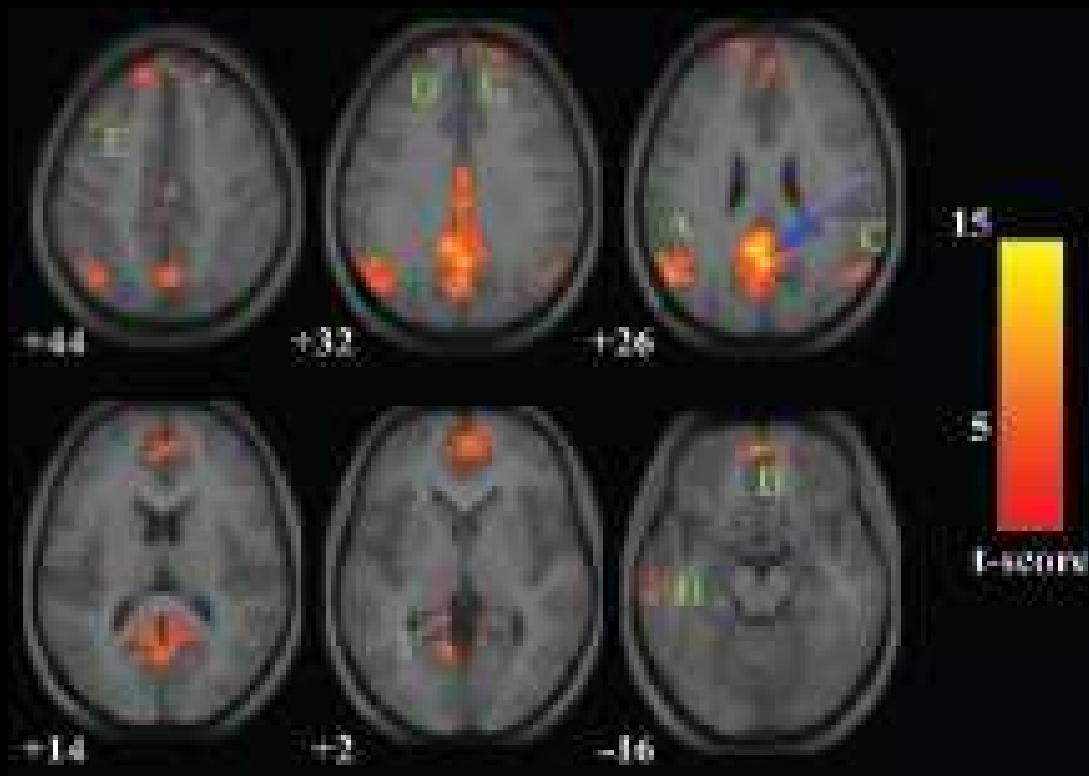
# Identification of a large-scale functional network: motor task



# Identification of a large-scale functional network: default mode hypothesis

**Functional connectivity in the resting brain : A network analysis of the default mode hypothesis.** Creicius et al. PNAS, 2003, 2004.

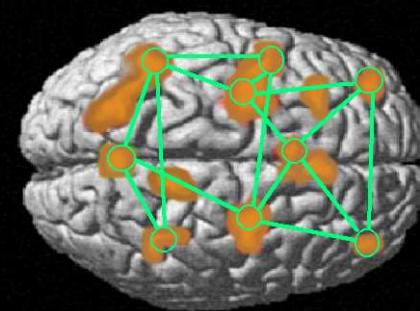
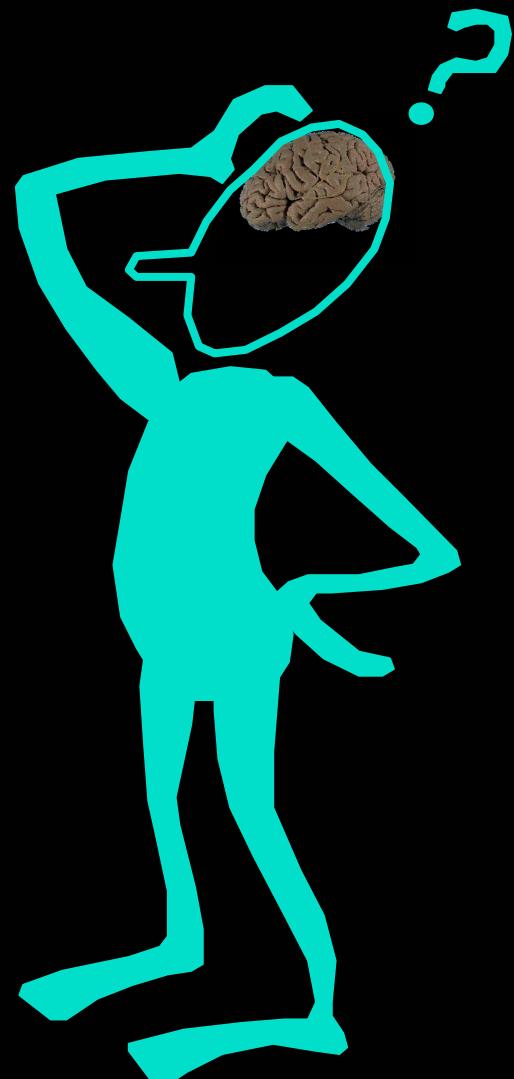
**ISNI model.** Bellec et al. Neuroimage, 2005



# Functional brain interactivity

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Functional connectivity



# Functional brain interactivity : non-linear correlation

Non linear correlation coefficient

$$h_{ij}^{2*} = \max_{\tau} h_{ij}^2(\tau).$$

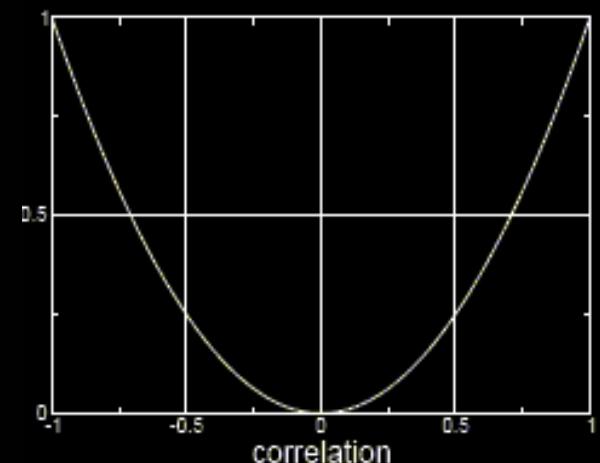
$$h_{ij}^2(\tau) = 1 - \frac{\text{Var}[Y_j(t + \tau) - h_{\hat{\theta}_{ij}(\tau)}(Y_i(t))]}{\text{Var}[Y_j(t + \tau)]}.$$

$$\hat{\theta}_{ij}(\tau) = \arg \min_{\theta} \mathbb{E} \left[ (Y_j(t + \tau) - h_{\theta}(Y_i(t)))^2 \right]$$

i.i.d Gaussian process ( $\mu$ ,  $\Sigma$ )

$$h_{\theta}(y) = \mu_j + a + b(y - \mu_i)$$

$$h_{ij}^{2*} = \rho_{ij}^2$$



# Functional brain interactivity : mutual information

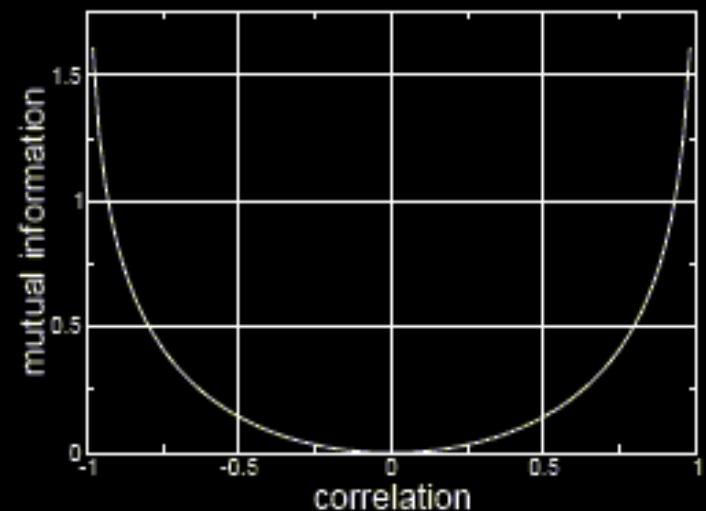
Mutual information

$$\text{MI}_{ij} = H(Y_i(t)) + H(Y_j(t)) - H(Y_i(t), Y_j(t))$$

$$H(X) = - \int \Pr(X) \cdot \ln \Pr(X) \, dX$$

i.i.d Gaussian process ( $\mu$ ,  $\Sigma$ )

$$\text{MI}_{ij} = -\frac{1}{2} \ln (1 - \rho_{ij}^2)$$



# Functional brain interactivity : Phase synchronization and PLV

Phase synchronization

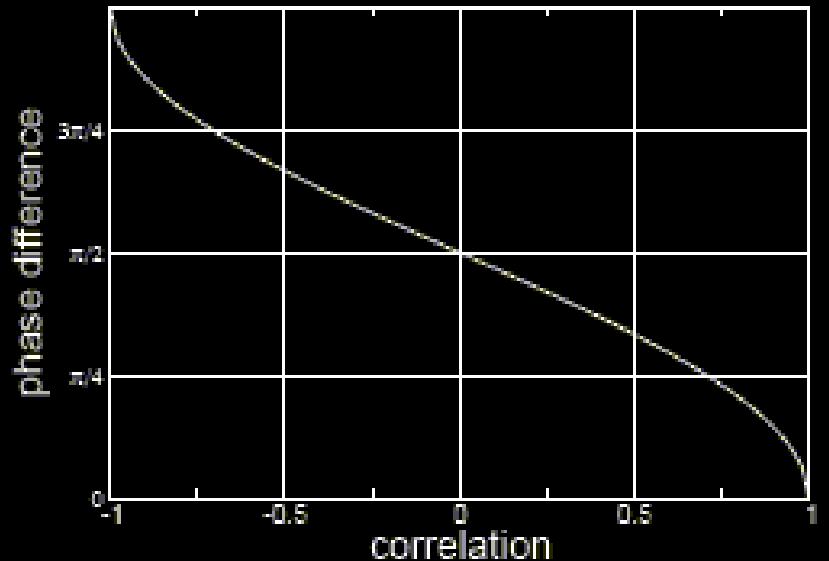
$$Y_n(t) = a_n \cos(2\pi\nu_n t + \phi_n)$$

$$\psi_{ij} = \phi_i - \phi_j$$

$$\boxed{\psi_{ij} = \arccos(\rho_{ij})}$$

$$\begin{aligned} E[\psi_{ij}] &= \int \psi_{ij} \cdot f(\psi_{ij}) d\psi_{ij} \\ &= \int \arccos(\rho_{ij}) \cdot g(\rho_{ij}) d\rho_{ij} \end{aligned}$$

i.i.d Gaussian process  $(\mu, \Sigma)$



Phase Looking Value

$$\gamma_{ij}^2(\nu) = \frac{\left| \frac{1}{E} \sum_{e=1}^E S_{i,e}(\nu) S_{j,e}(\nu)^* \right|^2}{\left[ \frac{1}{E} \sum_{e=1}^E S_{i,e}(\nu) S_{i,e}(\nu)^* \right]^2 \left[ \frac{1}{E} \sum_{e=1}^E S_{j,e}(\nu) S_{j,e}(\nu)^* \right]^2}$$

$$\boxed{\gamma_{ij}^2(\nu) = \left[ \int \cos(\psi_{ij}) \cdot f(\psi_{ij}) d\psi_{ij} \right]^2 + \left[ \int \sin(\psi_{ij}) \cdot f(\psi_{ij}) d\psi_{ij} \right]^2}$$

# Functional brain interactivity : Generalized synchronization

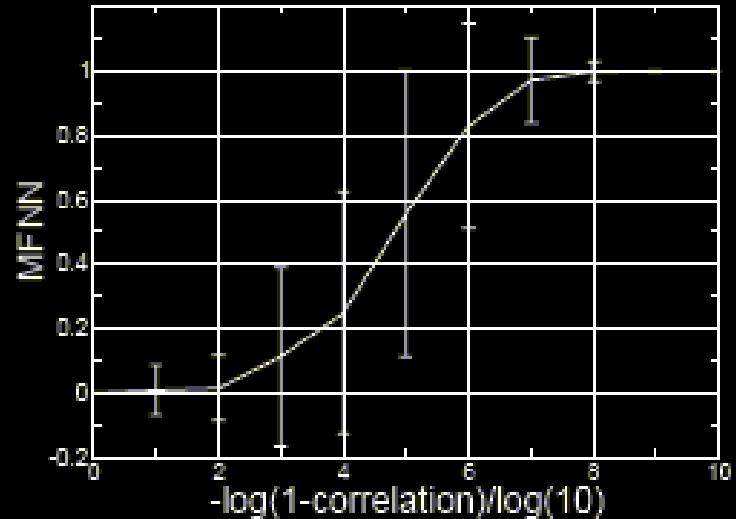
Generalized synchronization

$$\text{MFNN}_{ij} = \frac{|Y_i(t) - Y_i(\tau_{i,t})|}{|Y_j(t) - Y_j(\tau_{i,t})|} \cdot \frac{|Y_j(t) - Y_j(\tau_{j,t})|}{|Y_i(t) - Y_i(\tau_{j,t})|}$$

$$\tau_{n,t} = \arg \min_u |Y_n(t) - Y_n(u)|$$

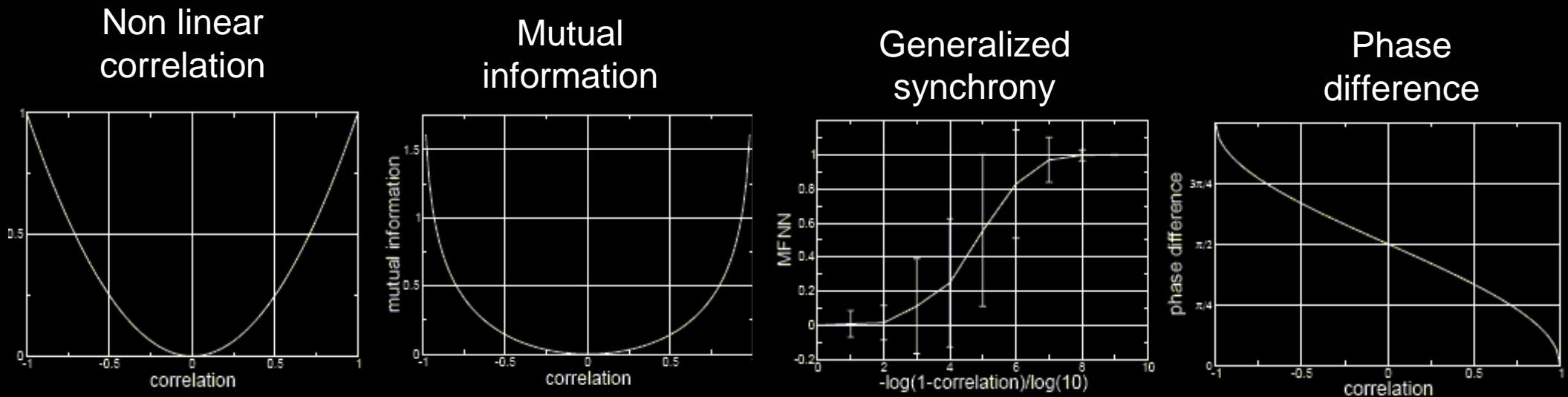
i.i.d Gaussian process ( $\mu$ ,  $\Sigma$ )

$$\Sigma = \begin{pmatrix} 1 & \rho \\ \rho & 1 \end{pmatrix}$$



# Functional brain interactivity : summary

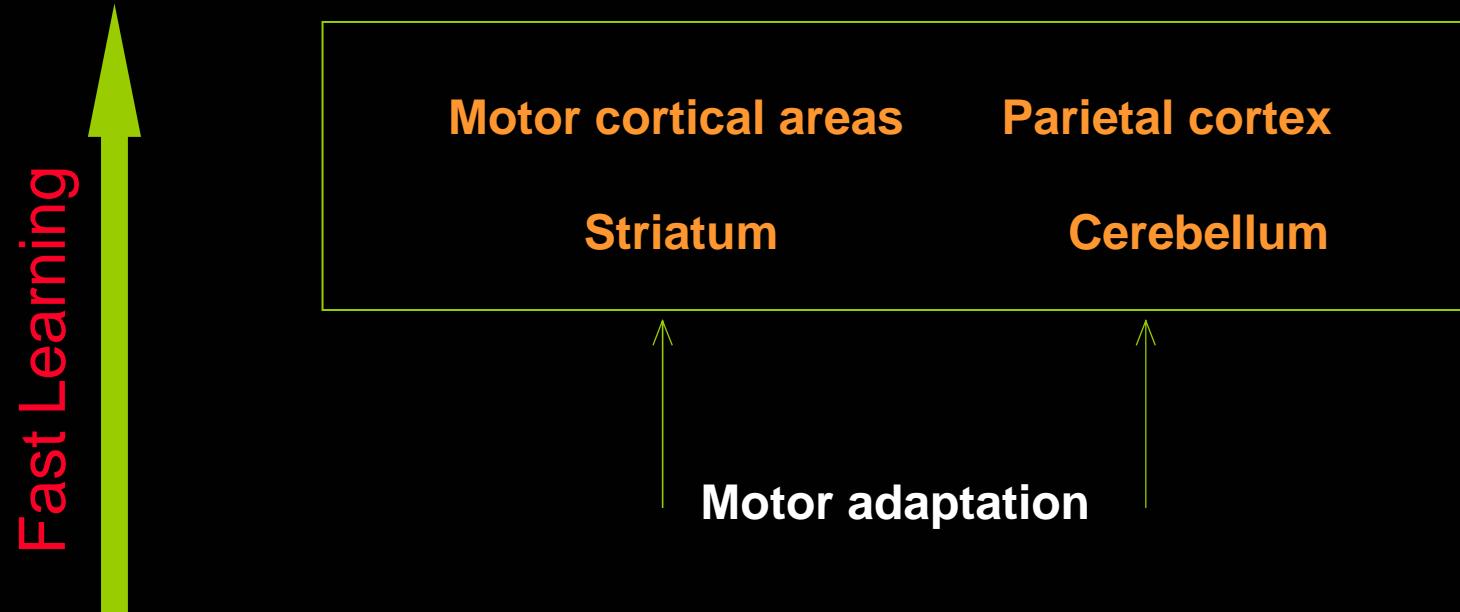
Hypothesis:  $Y$  i.i.d multivariate Gaussian, with mean  $\mu$  and covariance matrix  $\Sigma$



measure of interaction	expression	no interaction	maximum interaction
correlation	$\text{Corr}[Y_i, Y_j] = \rho_{ij}$	0	$\pm 1$
nonlinear correlation	$h_{ij}^{2*} = \rho_{ij}^2$	0	1
mutual information	$\text{MI}_{ij} = -\frac{1}{2} \ln (1 - \rho_{ij}^2)$	0	$+\infty$
generalized synchronization	?	0 or $+\infty$	1
phase difference	$\psi_{ij} = \arccos(\rho_{ij})$	$\pi/2 + k\pi$	$0 + k\pi$

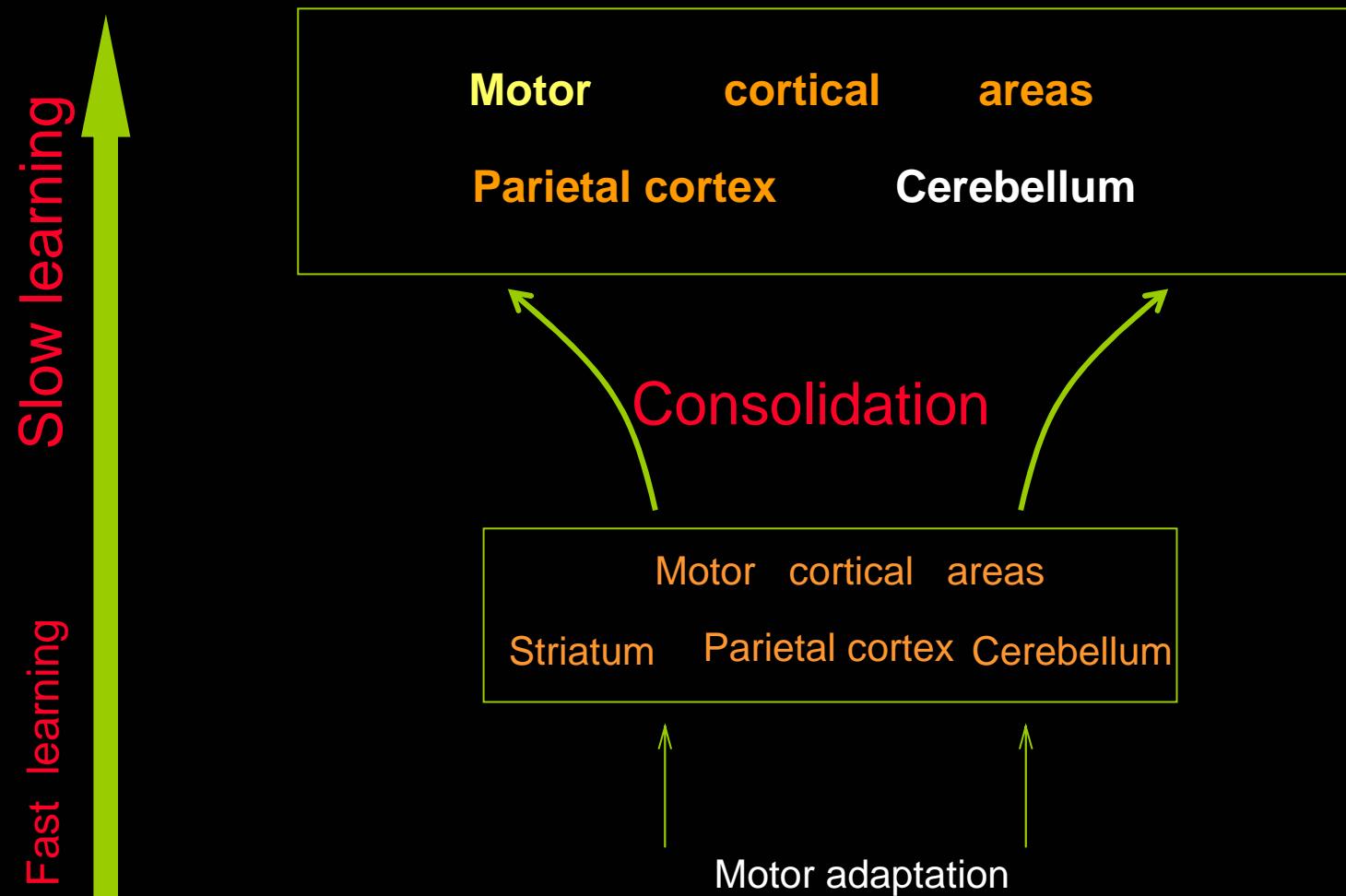
# Functional network : cognitive model

J. Doyon and L. Ungerleider's model



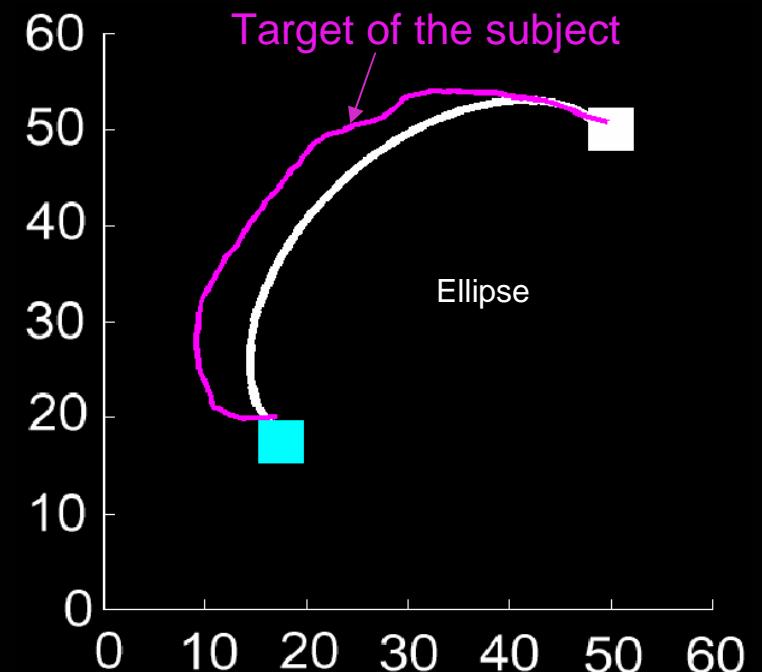
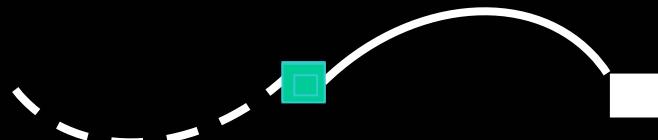
# Functional network : cognitive model

J. Doyon and L. Ungerleider's model



# Functional network : visuomotor adaptation task

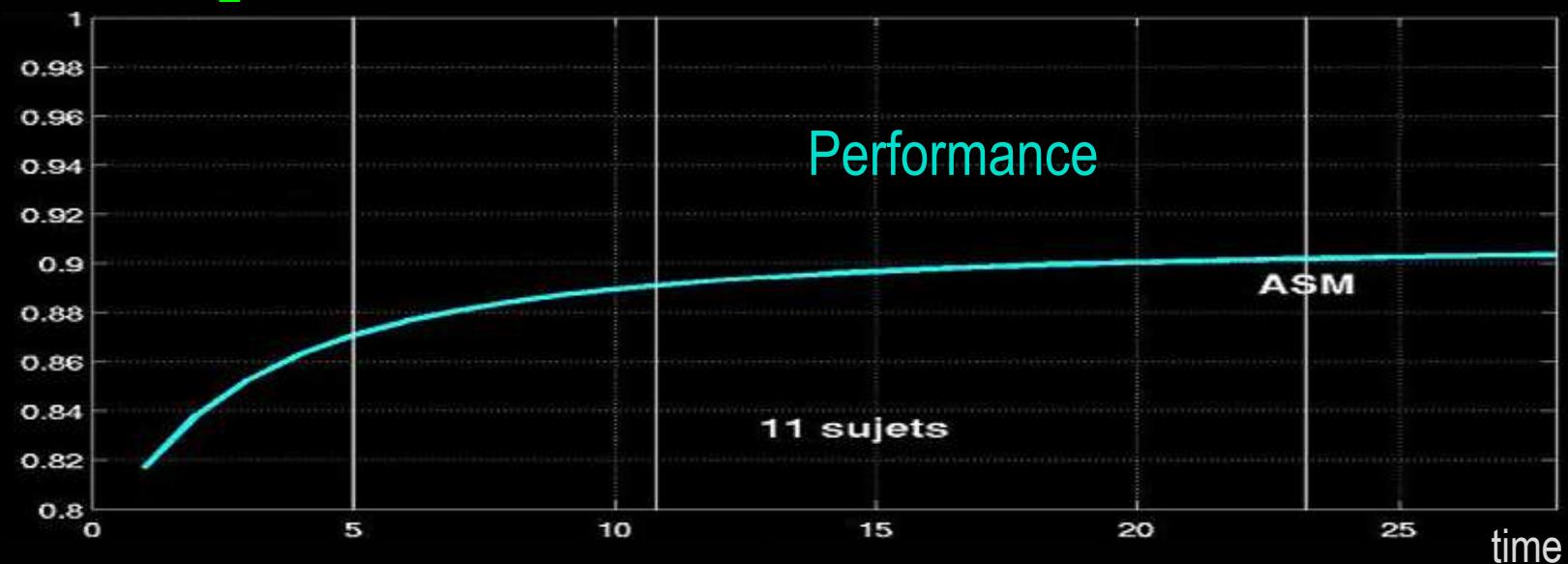
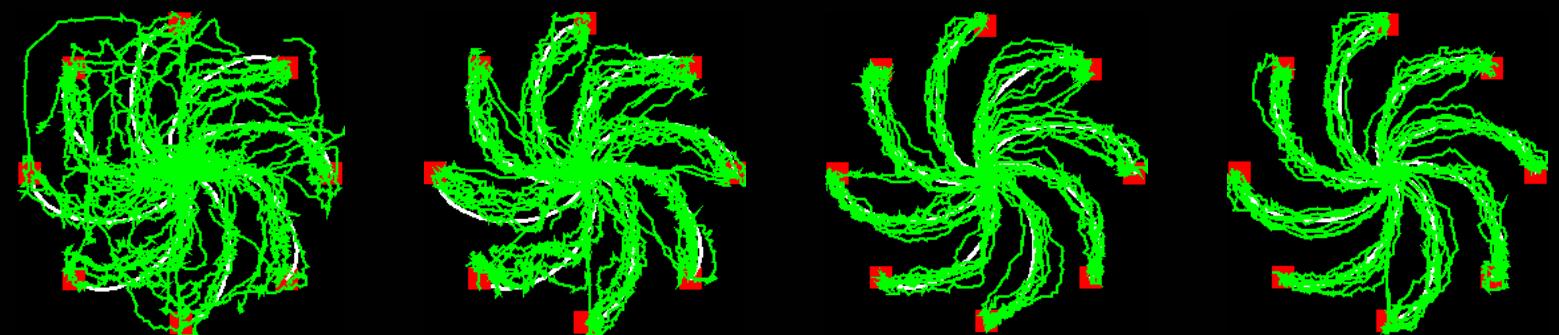
Visuomotor adaptation ASM Condition 'Indirect mode'



# Functional network : performance

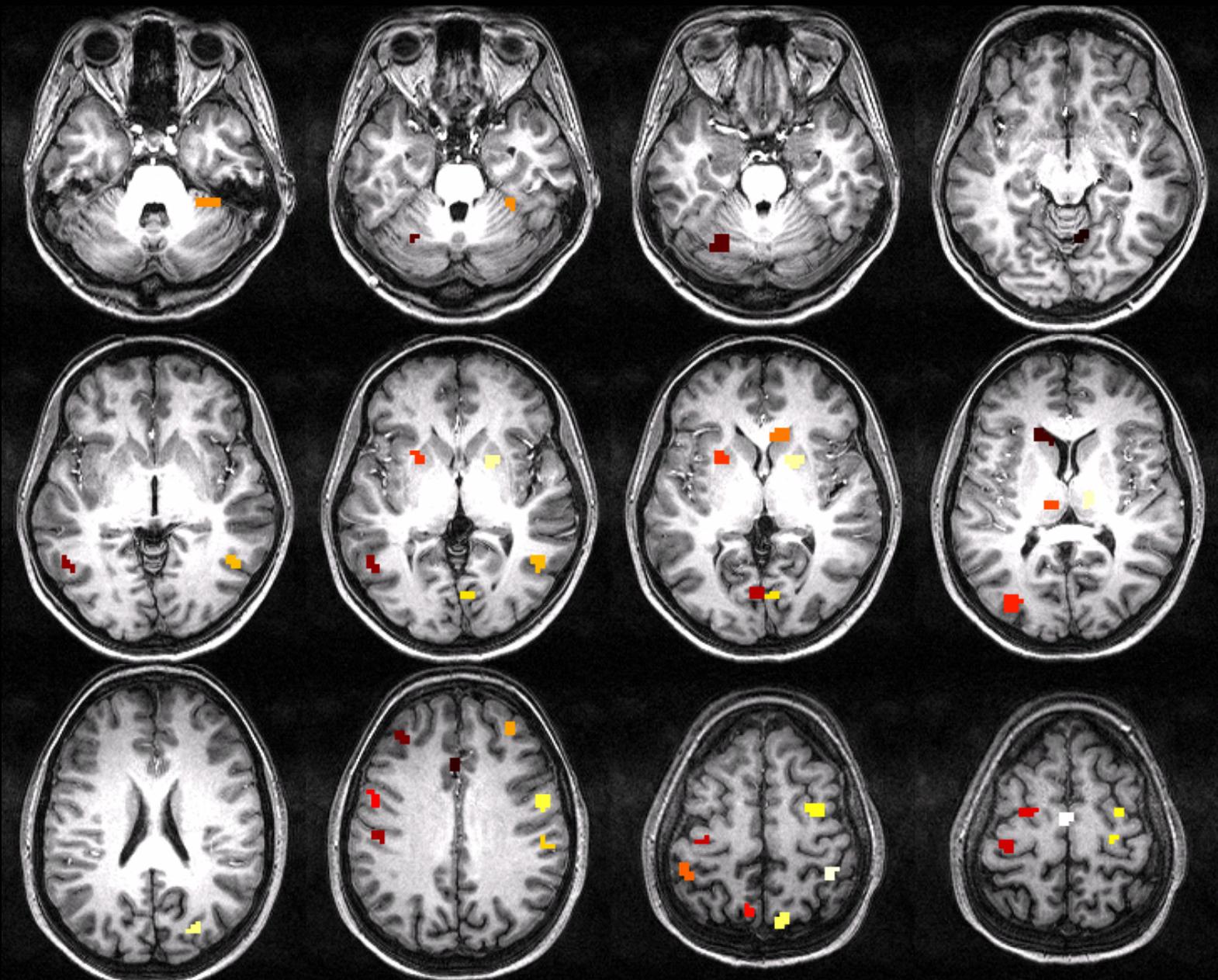


Does performance improvement reflect changes in functional connectivity?



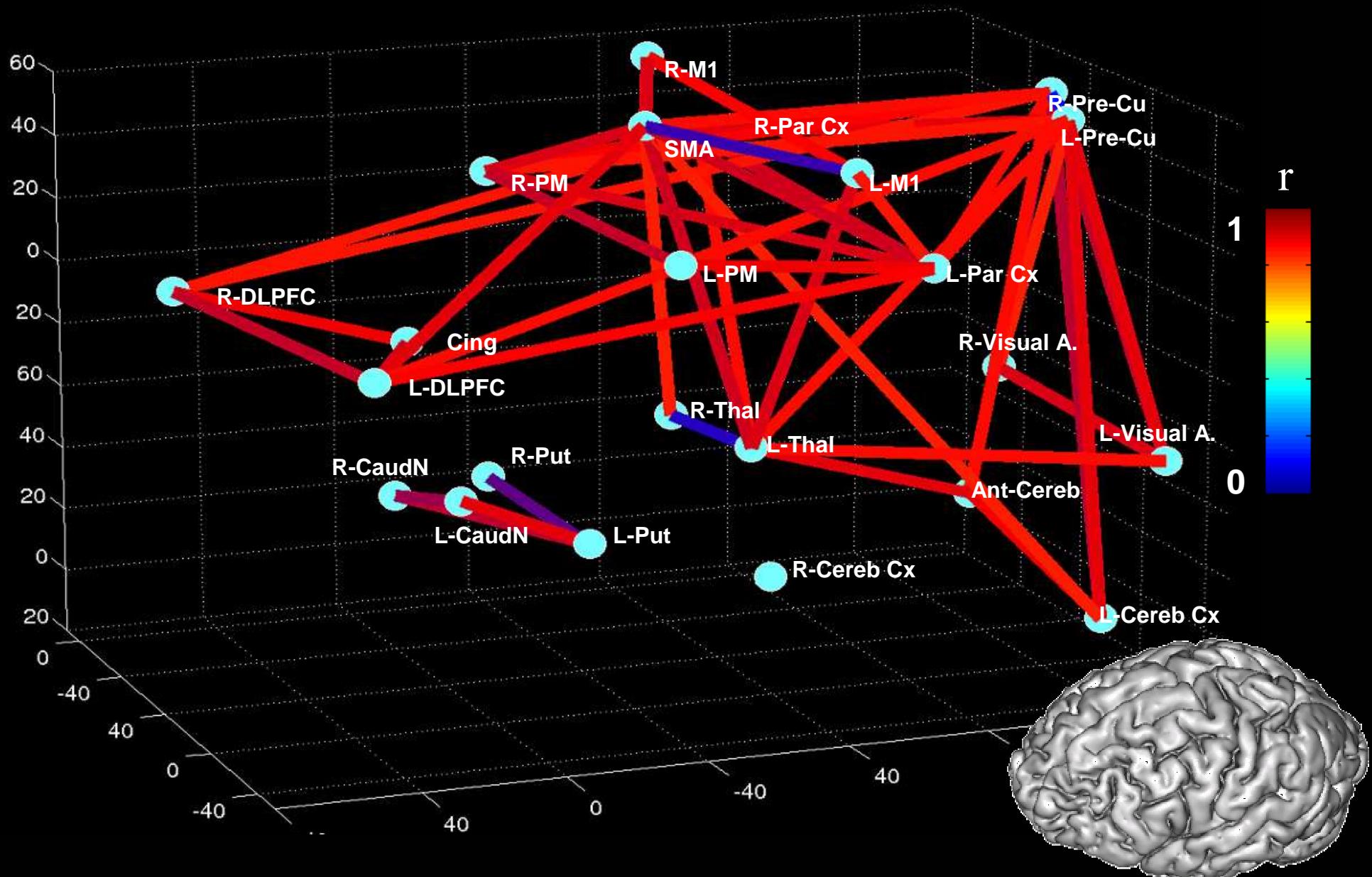
## Functional network : activation network

Brain units  
(15 voxels)



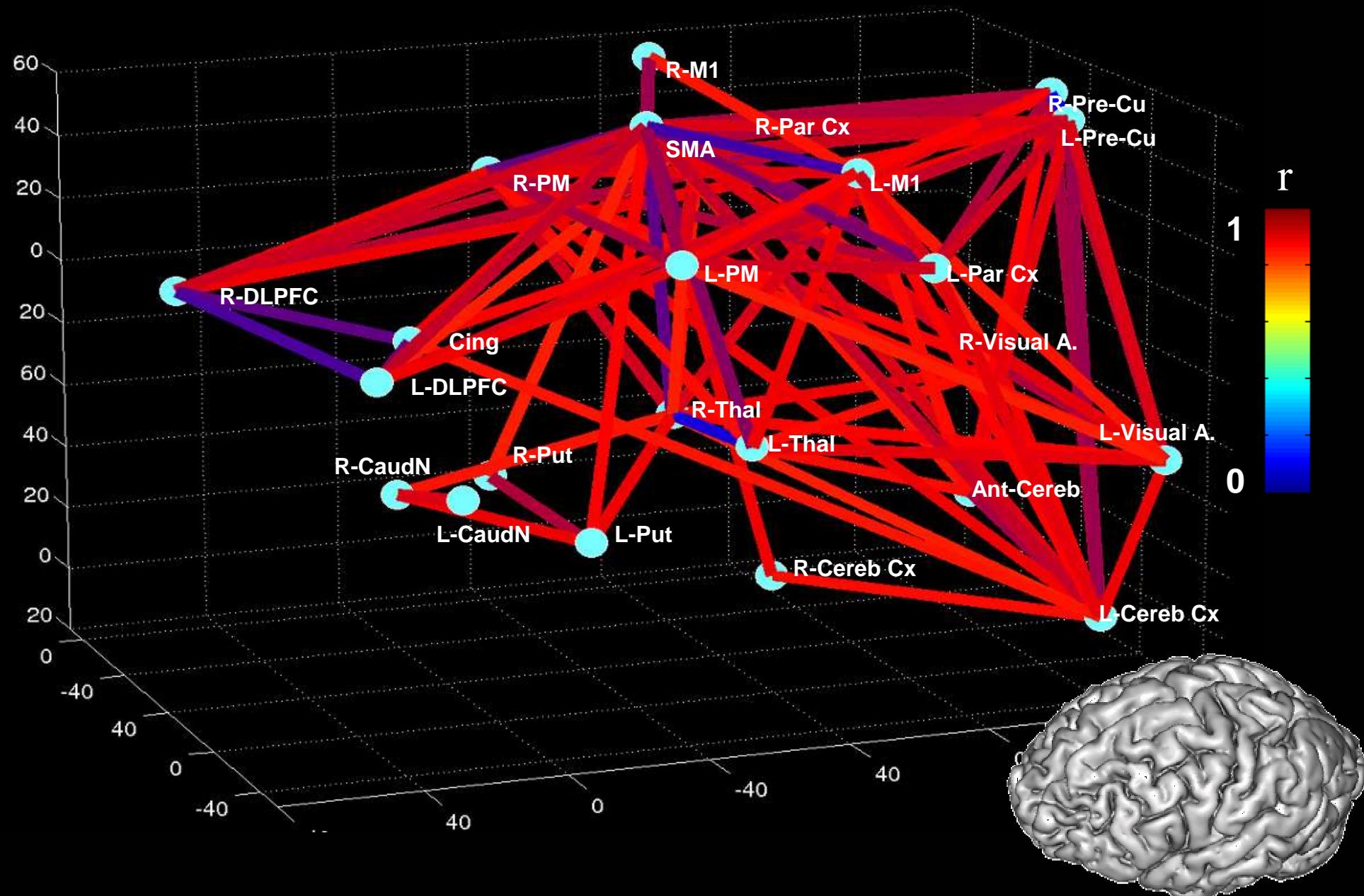
# Functional network

**Block 1 ( $p<0.01$ )**



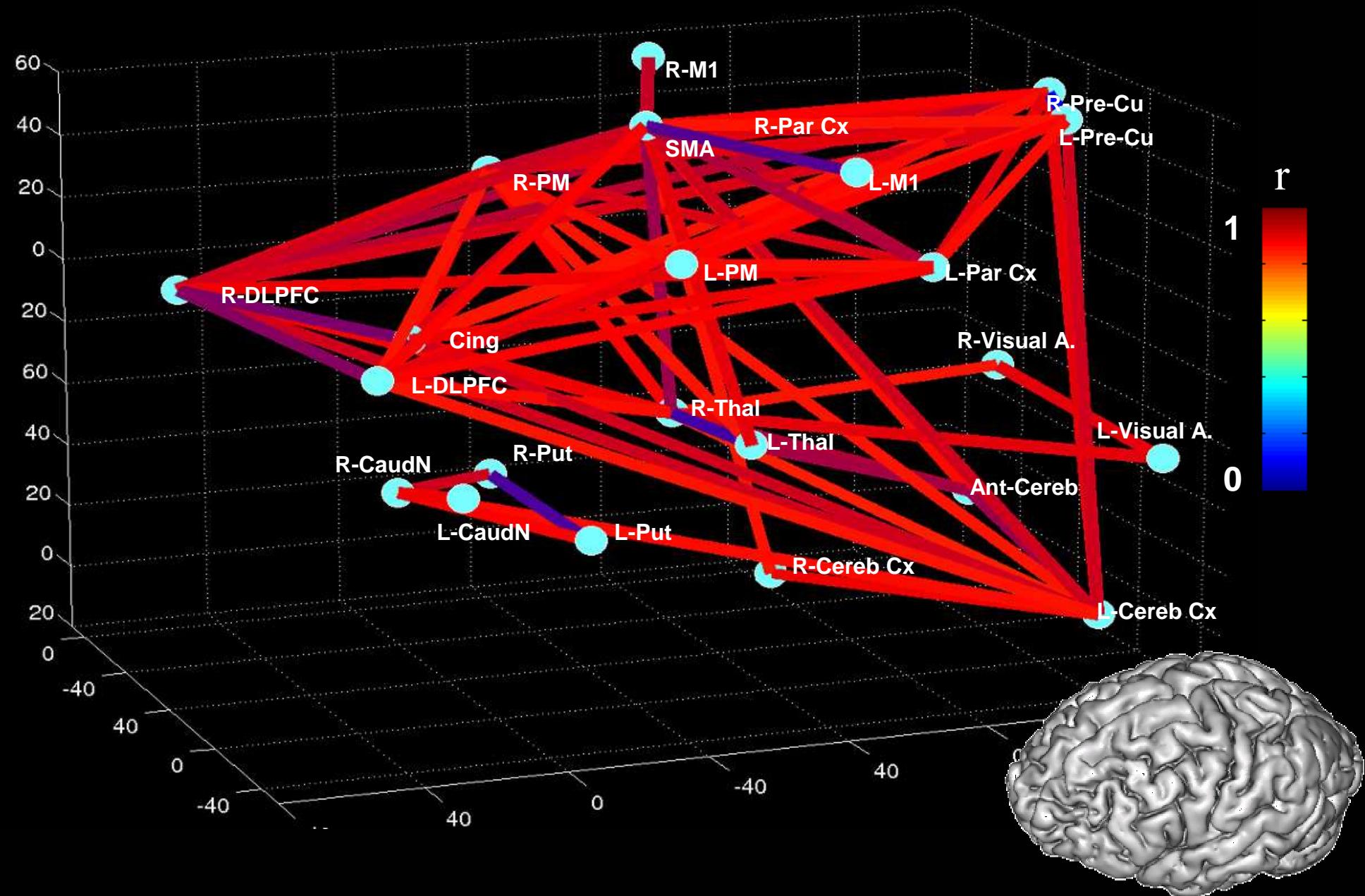
# Functional network

**Block 2 ( $p < 0.01$ )**



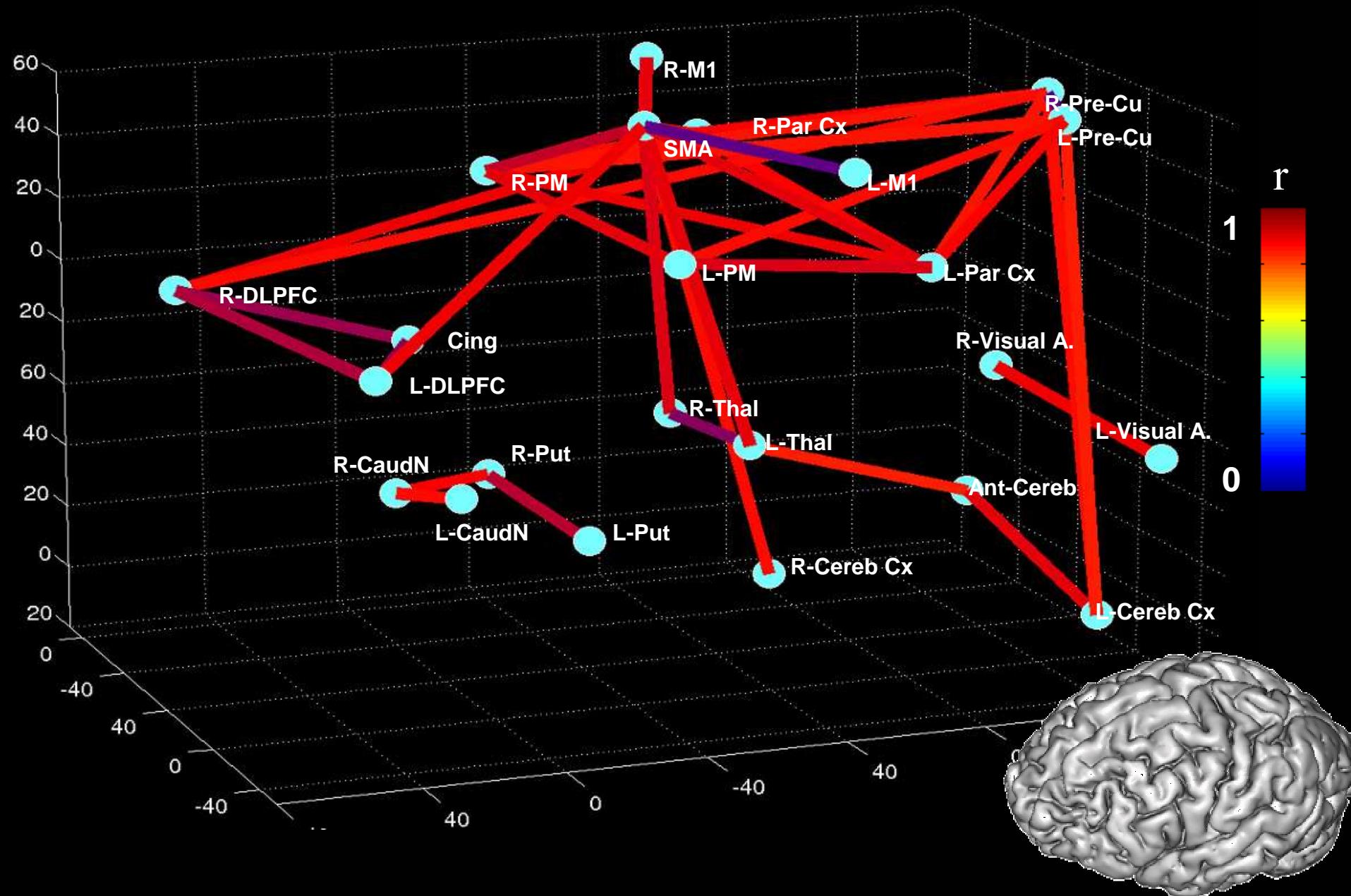
# Functional network

**Block 3 ( $p<0.01$ )**



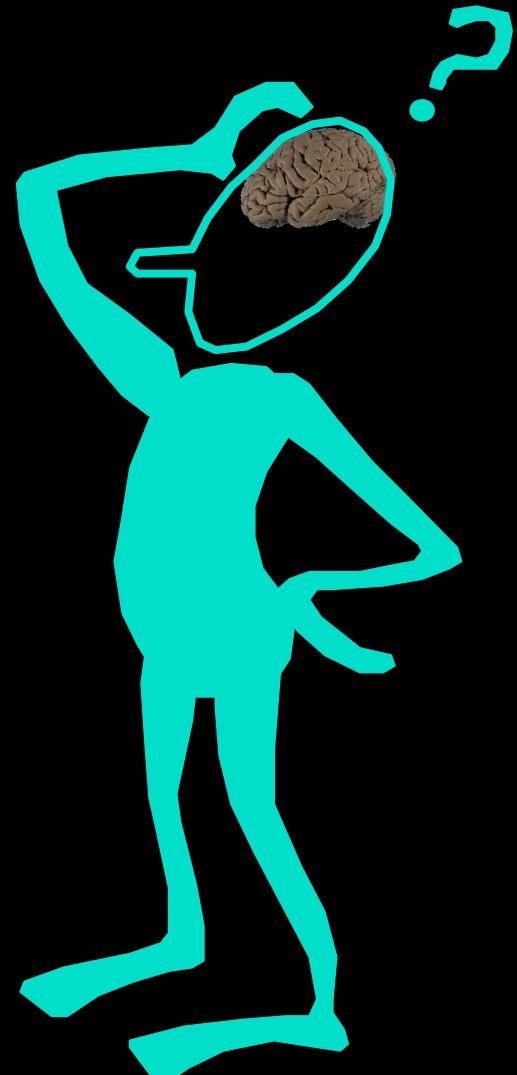
# Functional network

**Block 4 ( $p<0.01$ )**



# Dynamical functional connectivity

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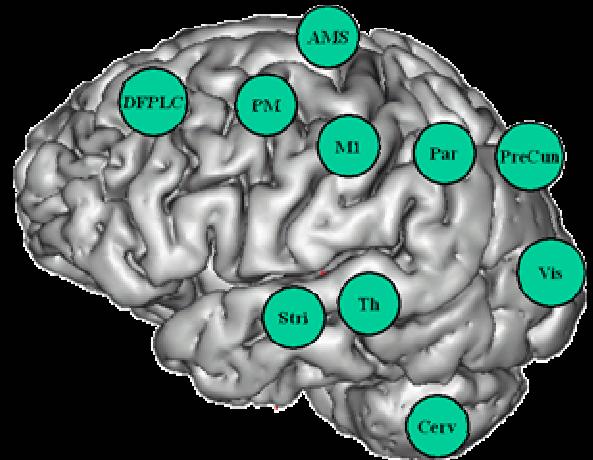


'The graphs are complex'

Global interaction effect: increasing  
then decreasing number of  
interactions

# Dynamical functional connectivity: Testing for global interaction over time

$$n_t = \text{Number } \{(i,j); \rho_{ij} > 0.333\} \text{ at } t$$



$$\text{Data} \sim N(\mu, \Sigma)$$

$$\text{Covariance } \Sigma \sim \text{Inv-Wishart}(v, S)$$

## Resampling method

1. Simulate  $B (\sim 1000)$  random variable  $\Sigma_t$  using the Inv-Wishart distribution and construct a correlation matrix  $\rho_t$ . Construct a random variable  $n_t$
2. The empirical distribution function  $n_{1t}, n_{2t}, \dots, n_{Bt}$  approximates the distribution of  $n_t$  for large  $B$ .

# Dynamical functional connectivity: Testing for global interaction over time

# connections

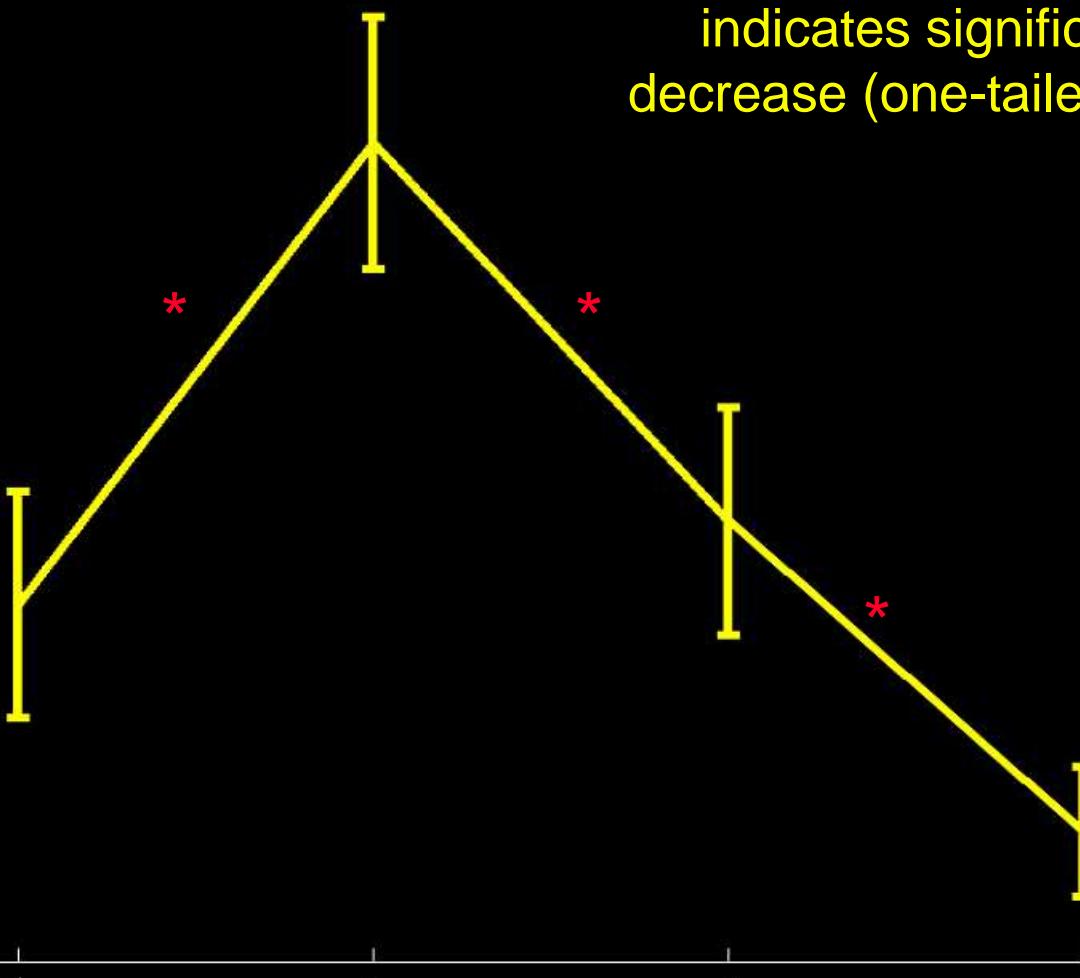
240  
220  
200  
180  
160  
140  
120  
100  
80  
60

1 Day 1 2

3 Day 2 4

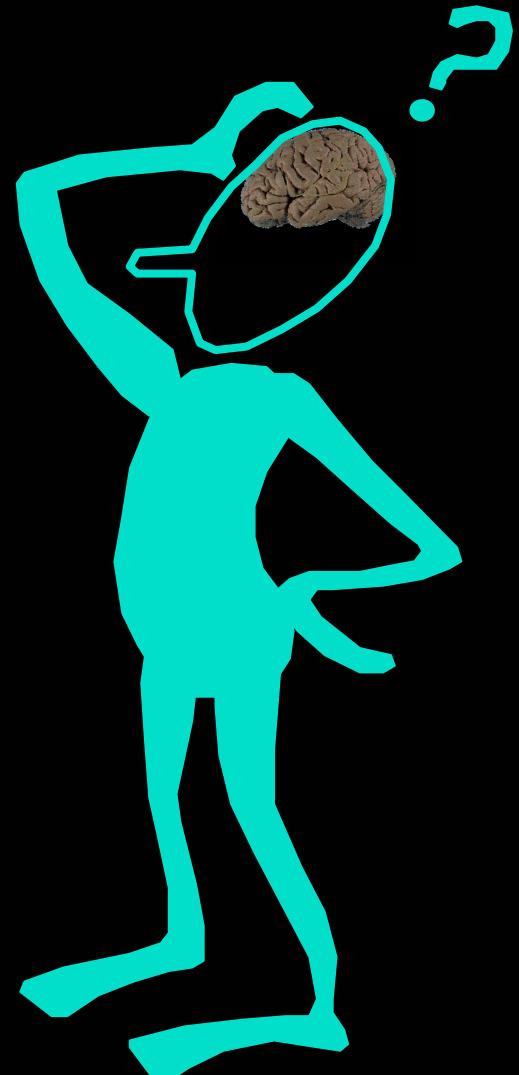
block

Number of connections per block. A \* indicates significant increase or decrease (one-tailed  $\zeta$ -test,  $p < 0.05$ ).



# Dynamical functional connectivity

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Quantifying changes of interaction  
between areas and relating it to the  
cognitive model of motor adaptation

# Dynamical functional connectivity: Testing for dynamics

$$H_0 : \rho_{ij}(t) \neq 0 \text{ and } \rho_{ij}(t') \neq 0 \text{ and } \rho_{ij}(t) = \rho_{ij}(t')$$

## Statistical test

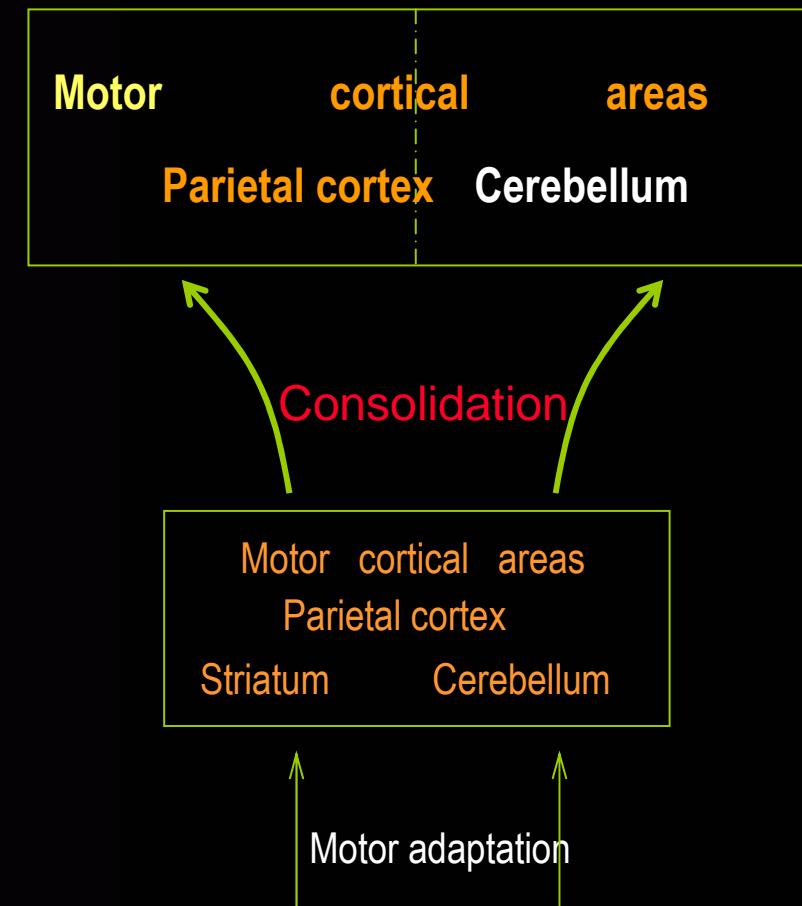
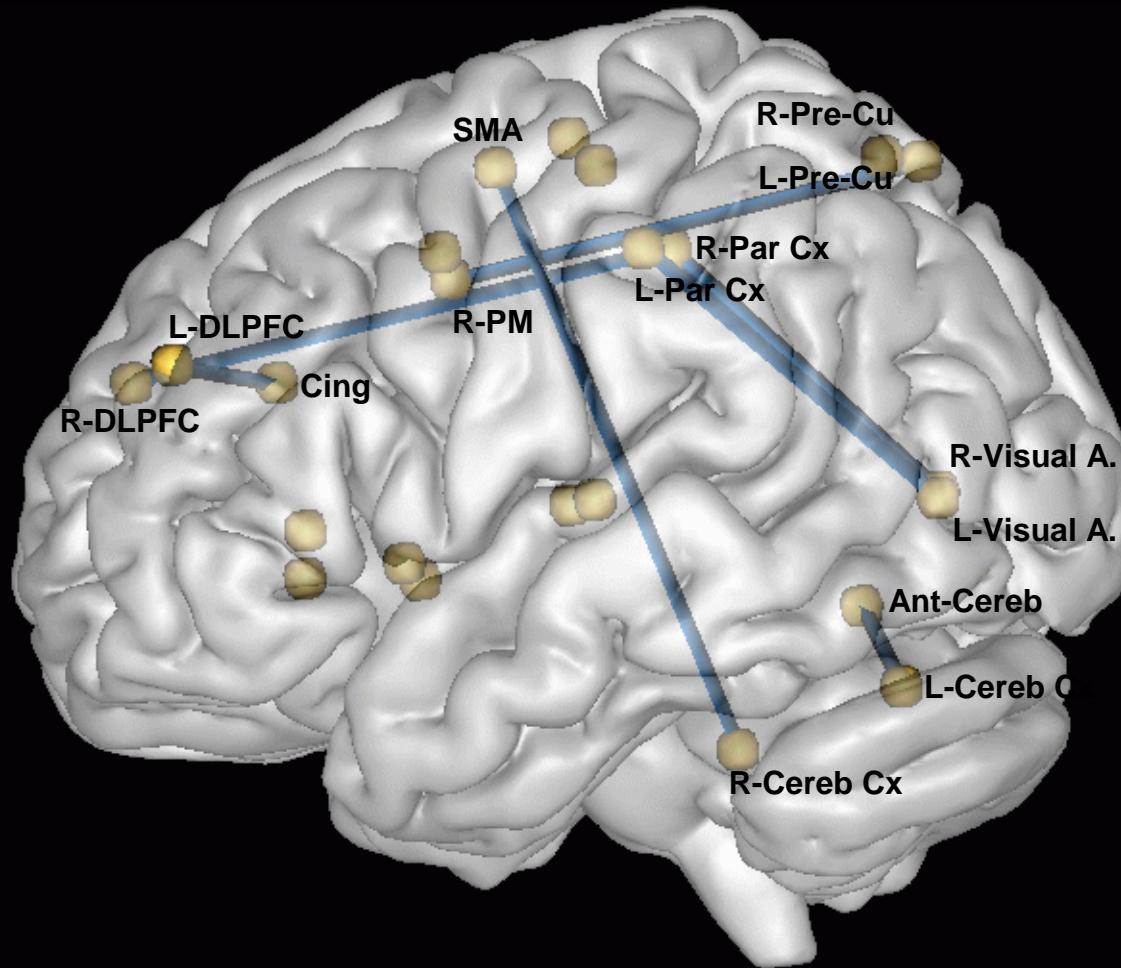
$$\zeta_{ij}(t,t') = \frac{\rho_{ij}(t) - \rho_{ij}(t')}{\sqrt{v(t) + v(t')}}$$

## Resampling method

1. Simulate  $B (\sim 1000)$  random variable  $S_t, t=1,T$  using the Inv-Wishart distribution and construct a correlation matrix  $\rho_t, t=1,T$ . Construct a random variable  $\zeta_{ij}(t,t')$
2. The empirical distribution function  $\zeta_{ij}^1(t,t'), \zeta_{ij}^2(t,t') \dots \zeta_{ij}^B(t,t')$  approximates the distribution of  $\zeta_{ij}(t,t')$  for large  $B$ .

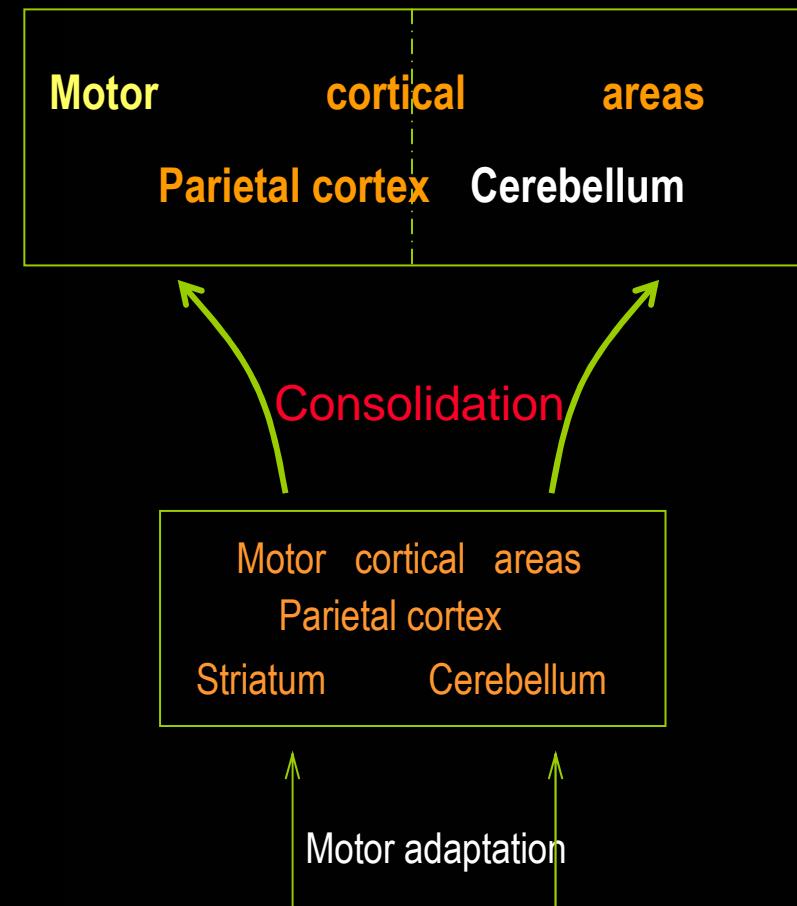
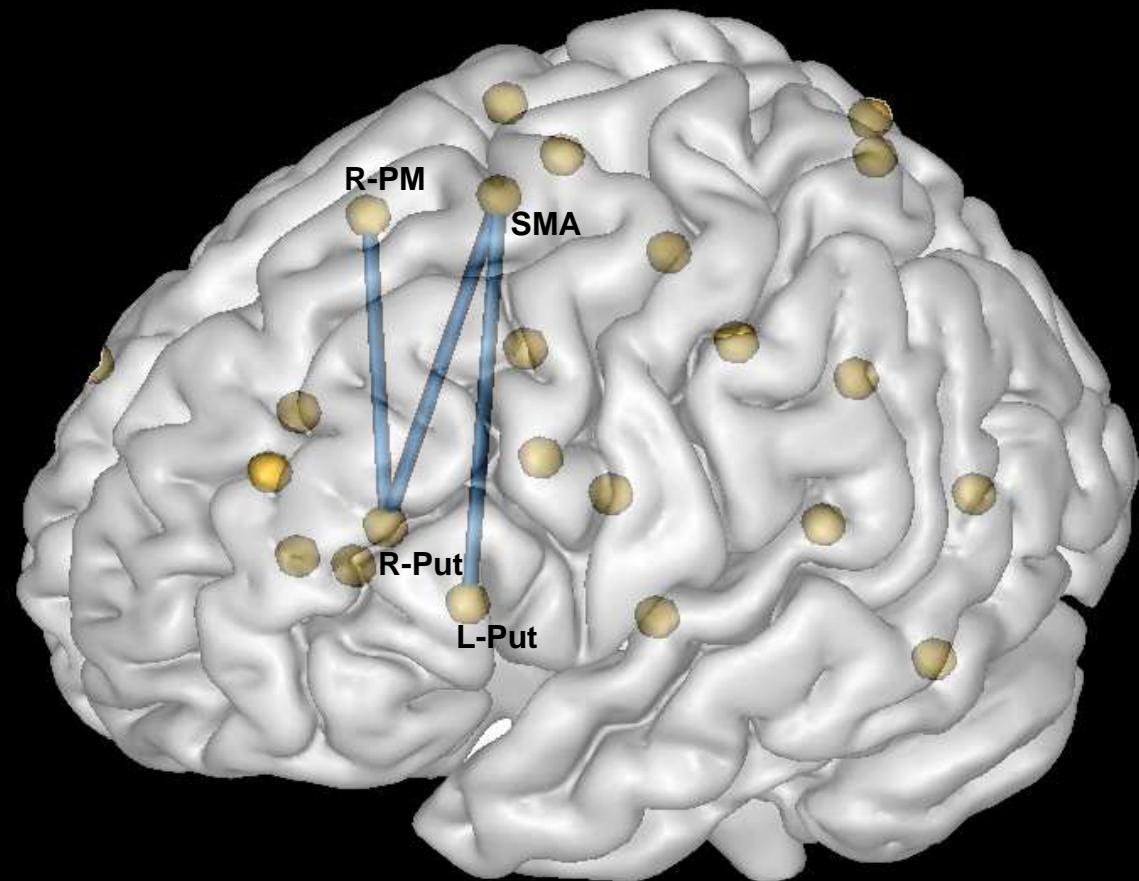
# Dynamical functional connectivity: Testing for dynamics

Specialization graph: connections significantly increased from block 1 to 2, not significantly decreased from block 2 to 4, and still significant in block 4 (one-tailed  $\zeta$ -test,  $p < 0.05$ )



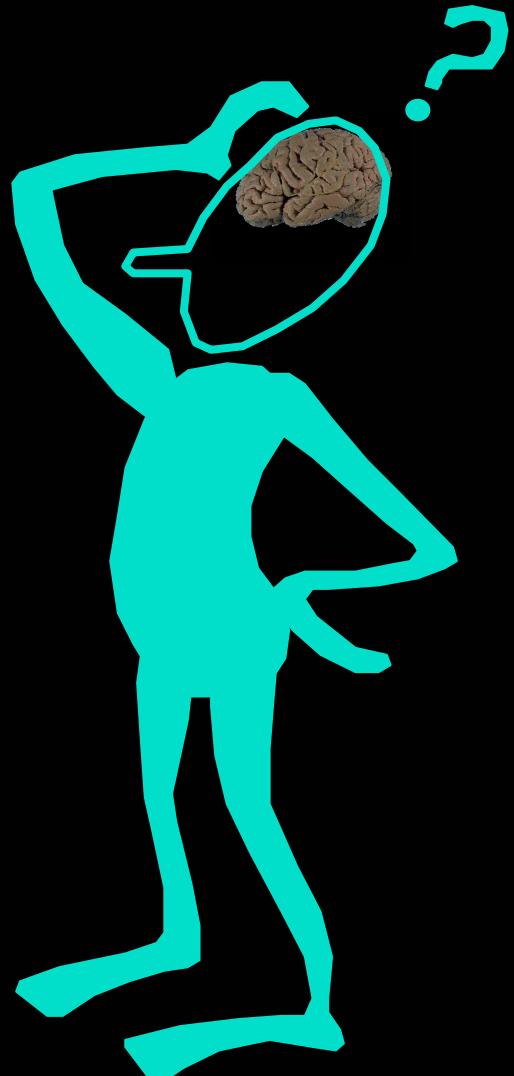
# Dynamical functional connectivity: Testing for dynamics

Specialization graph: connections significantly increased from block 1 to 2, significantly decreased from block 2 to 4, and not significant in block 4 (one-tailed  $\zeta$ -test,  $p < 0.05$ )



# Functional connectivity : direct or indirect interaction

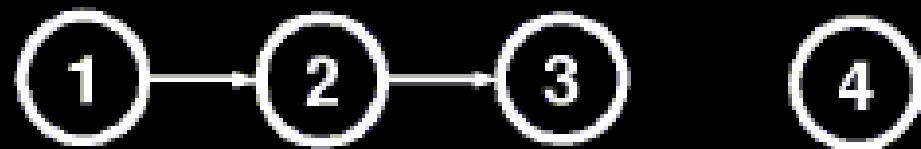
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Does interactivity reflect direct changes in functional connectivity ?

# Functional connectivity : Conditional interactivity

Gap between effective and correlational connectivity



Model

$$\begin{aligned} Y_1 &= & E_1 \\ Y_2 &= \lambda \cdot Y_1 & + E_2 \\ Y_3 &= & \mu \cdot Y_2 & + E_3 \\ Y_4 &= & & E_4 \end{aligned}$$

Effective connectivity

$$\text{Corr}[Y_1, Y_4] = 0$$

$$\text{Corr}[Y_1, Y_2] = \frac{\lambda}{\sqrt{\lambda^2 + 1}} \neq 0$$

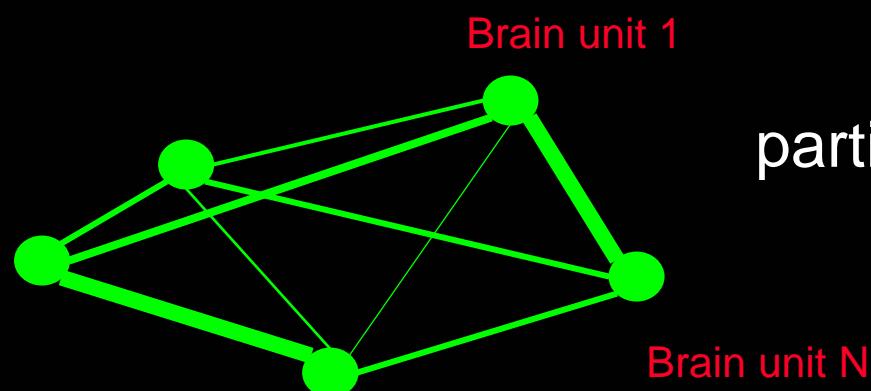
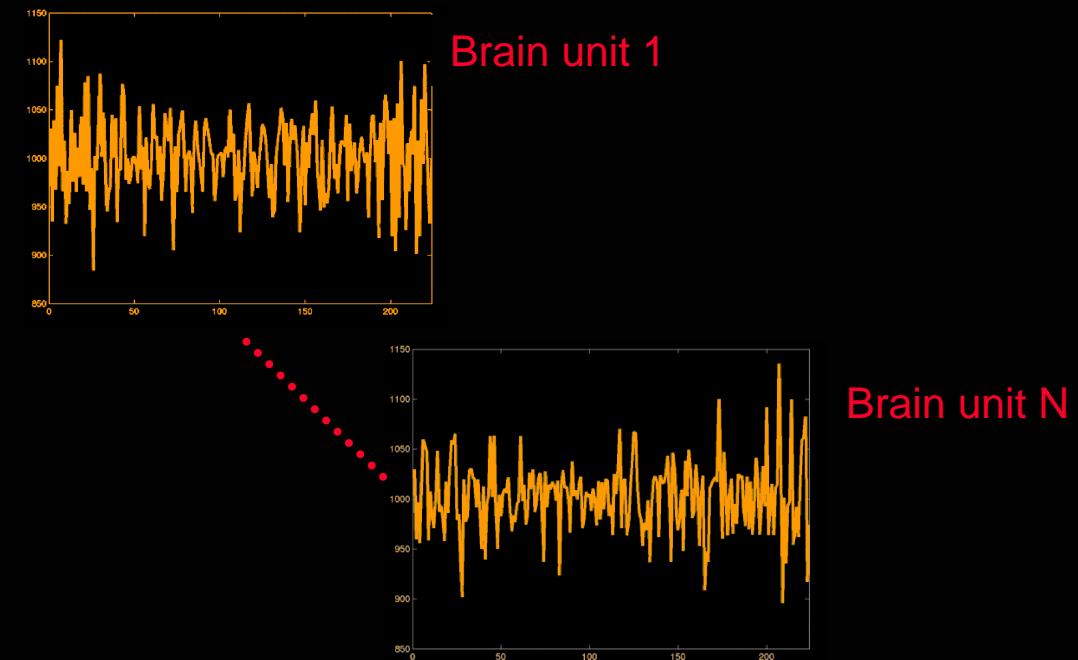
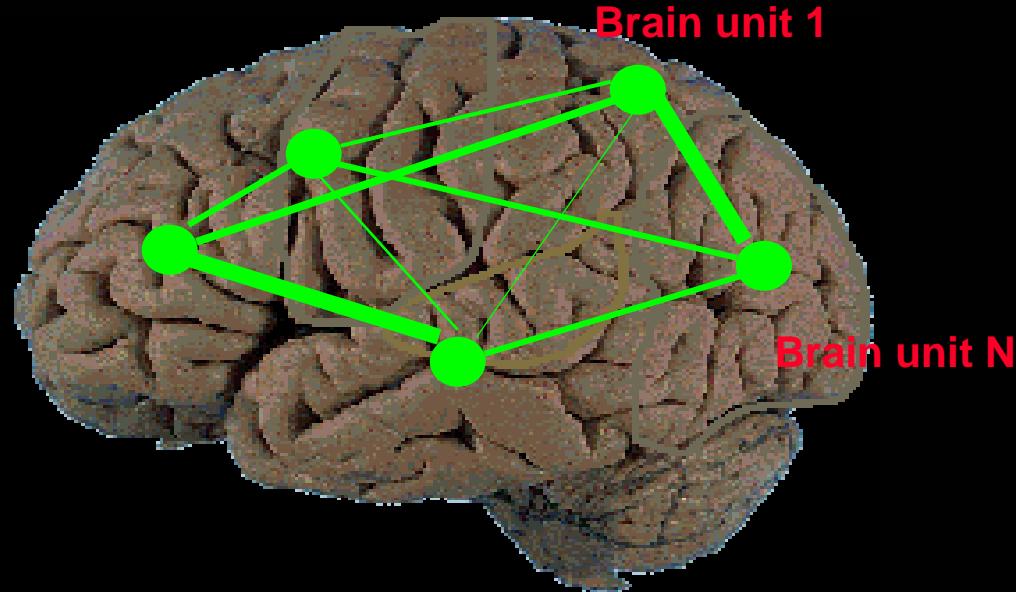
$$\text{Corr}[Y_1, Y_3] = \frac{\lambda\mu}{\sqrt{\lambda^2\mu + \mu}} \neq 0$$

Conditional correlation

$$\text{Corr}[Y_1, Y_2|Y_3] = \frac{\lambda}{\sqrt{(\lambda^2 + 1)(\mu^2 + 1)}} \neq 0$$

$$\text{Corr}[Y_1, Y_3|Y_2] = 0$$

# Functional connectivity : Conditional interactivity



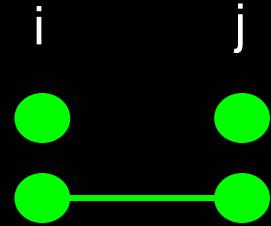
partial correlation  $\pi_{ij} \in [-1, 1]$   
 $\pi_{ij} = 0$  : no direct interaction  
 $\pi_{ij} \neq 0$  : not independent

# Functional connectivity : Conditional independence graph model

$$p(\gamma | H, y) \propto p(y | H, \gamma) \cdot p(\gamma | H)$$

(Bayesian approach)

graph model (qualitative)

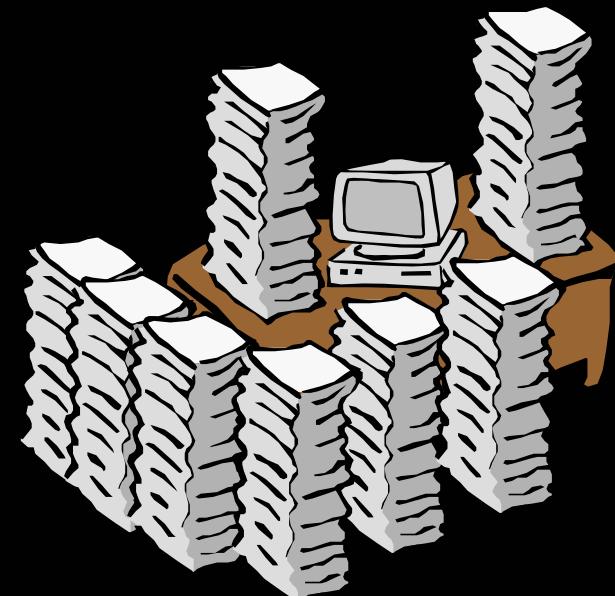


$$\begin{aligned}\gamma_{ij} = 0 &\Rightarrow \pi_{ij} = 0 \\ \gamma_{ij} = 1 &\Rightarrow \pi_{ij} \in [-1, 1]\end{aligned}$$

data model (quantitative)

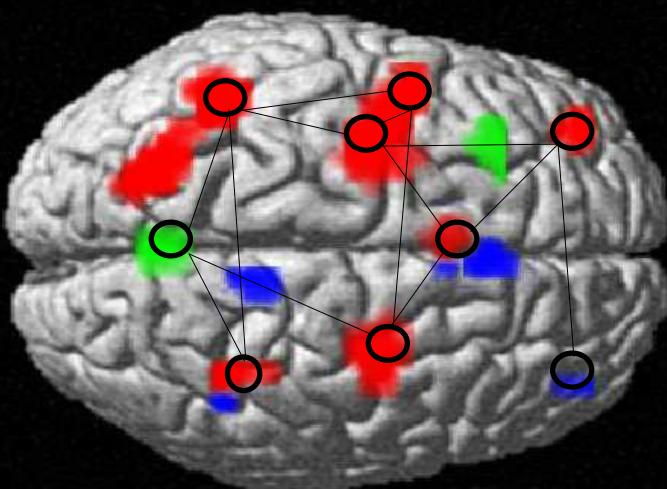
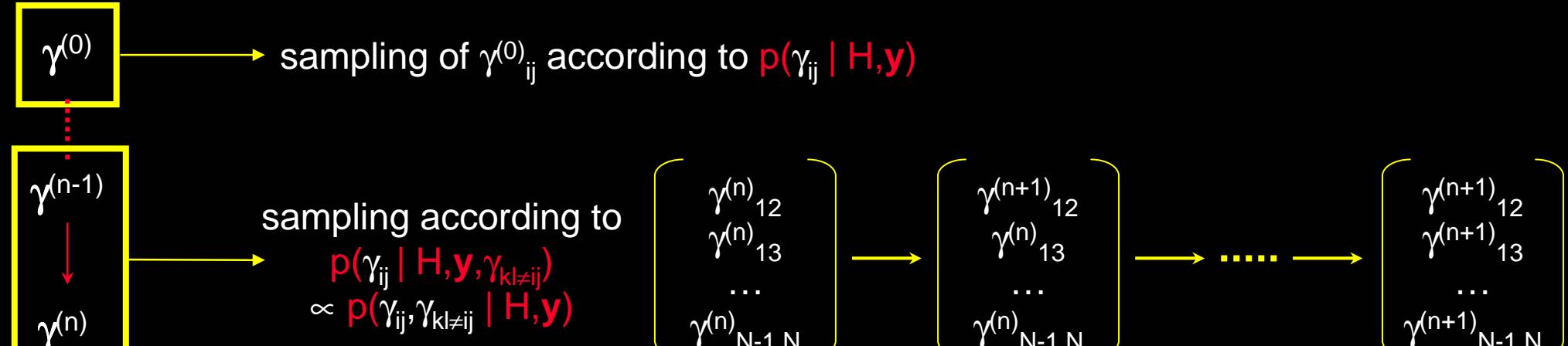
$$y_t | H, \pi \sim N(m, V)$$

variance  
mean



# Functional connectivity : Conditional independence graph model

$$p(\gamma | H, y) \propto p(\gamma | H) 2^{-l(\gamma)} N(\mathbf{m}_0, \mathbf{V}_{00}; 0_0)$$

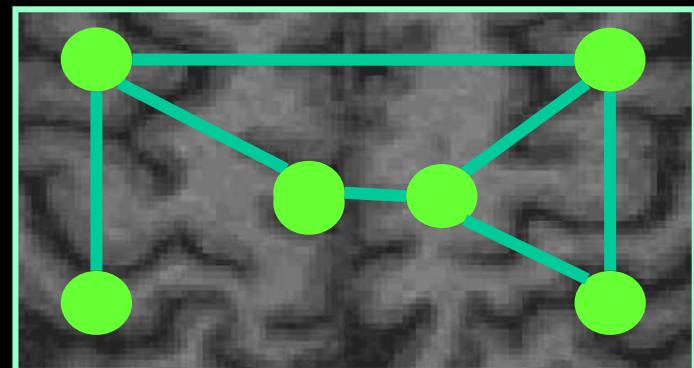


graph estimation:  $\gamma_e = \text{MAP}[p(\gamma | H, y)]$ ;  $\gamma_m = E[\gamma | H, y]$

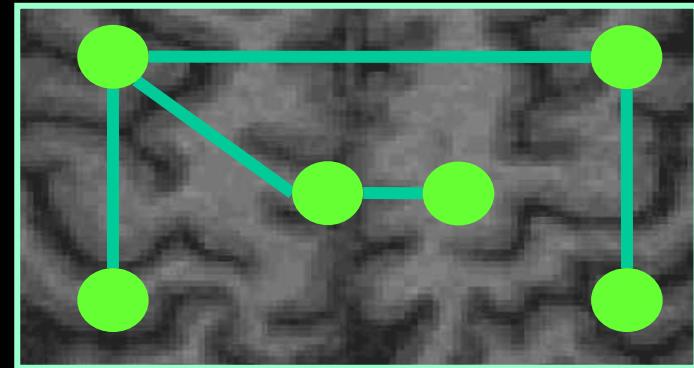
# Functional connectivity : Conditional independence graph model

## Simple motor task using fMRI

Right



Left



## Research group



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