

Ecole d'été Maths et Cerveau
Jeudi 16 juin 2005

**Probabilistic models of
3D shape and motion perception**

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« *Le hasard n'est que la mesure de notre ignorance* »

Henri Poincaré, *La science et l'hypothèse*, 1902

⇒ *Probability is the best way to quantify knowledge*

The problem ...



Deep Blue beats Kasparov (1997)



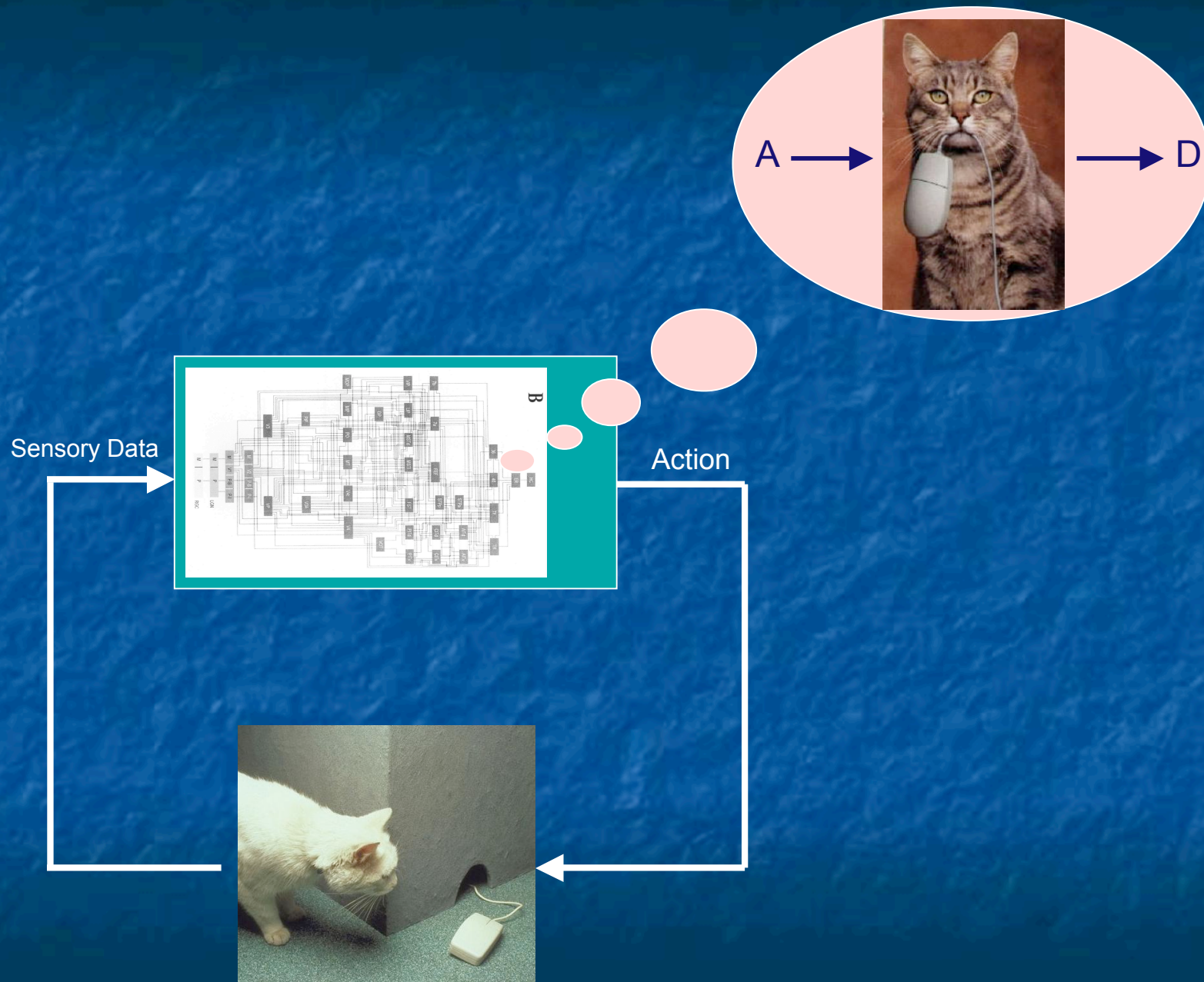
Children outmatch the most powerful machines in object perception

Imperfect knowledge about brain functions

Brain's imperfect knowledge about outside world:

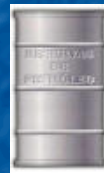
- uncertainty and incompleteness of sensory data
- limited actions, temporal constraints
- internal model incompleteness





How to express various forms of knowledge ?

generalization
↓



= 51,12 \$

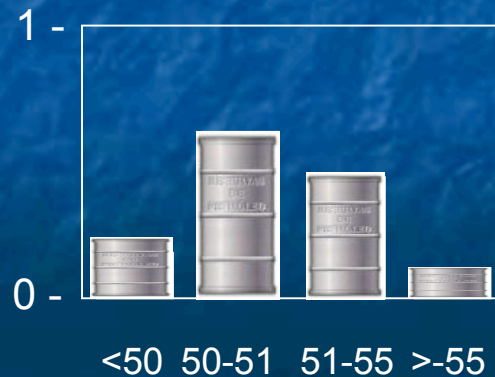
Ex.: Data, facts, ...



50 \$ <

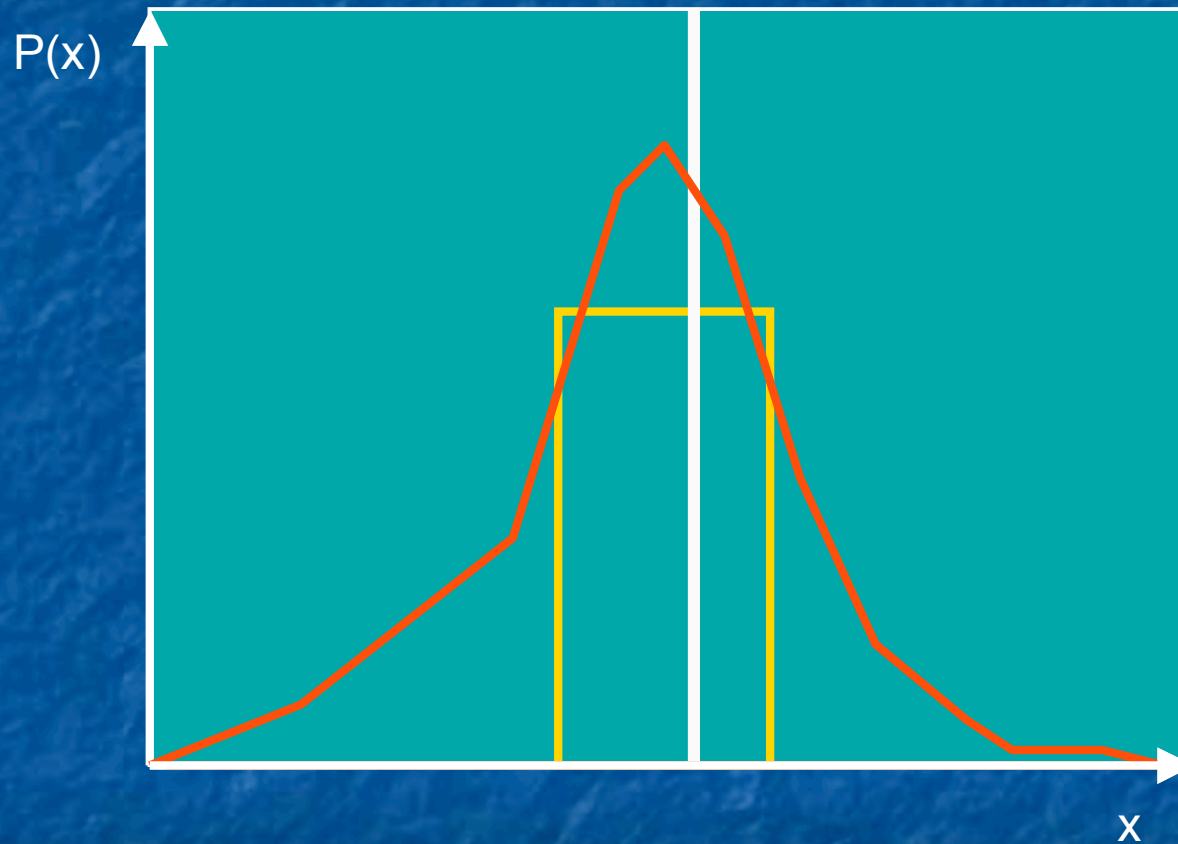
< 52 \$

Ex.: thresholds,
validity domains, ...



Ex.: uncertainty,
variability, ...

⇒ Probability distribution as a common language for all knowledge forms



$$x \in E \subseteq [0, 1]$$

$$\sum_x P(X) = 1$$

If ... Then ...



?



generalization

“If I know x , then y is exactly known” : function $x \rightarrow y = f(x)$

“If I know x , then y is confined in some subset” : $R(x,y) = \text{true}$

“If I know x , then I can improve my knowledge on y ”: $P(y | x) = P(x, y) / P(x)$

The 3 main steps:

1. Choice of variables :



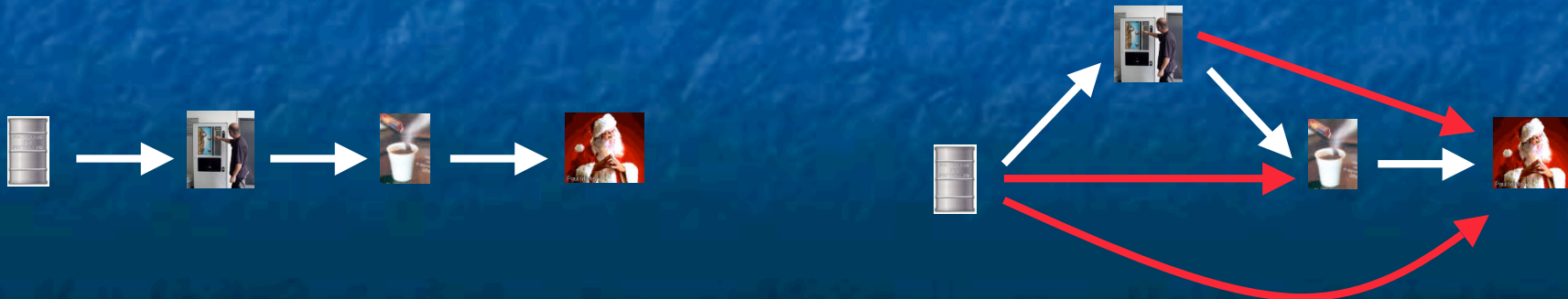
2. Expression of knowledge:

$$P(\text{Refrigerator}, \text{Milk}, \text{Vending Machine}, \text{Santa Claus}) =$$

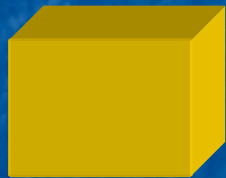
$$P(\text{Refrigerator}) \cdot P(\text{Vending Machine} \mid \text{Refrigerator}) \cdot P(\text{Milk} \mid \text{Vending Machine}) \cdot P(\text{Santa Claus} \mid \text{Milk})$$

3. Exploitation:

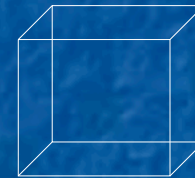
$$P(\text{Santa Claus} \mid \text{Refrigerator}) \sim \sum_{\text{Milk}, \text{Vending Machine}} P(\text{Refrigerator}, \text{Milk}, \text{Vending Machine}, \text{Santa Claus})$$



The perception is an inverse problem...



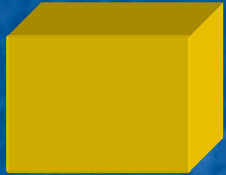
Physical State S



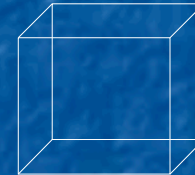
Observed sensory data D



Worse, it is an ill-posed inverse problem...



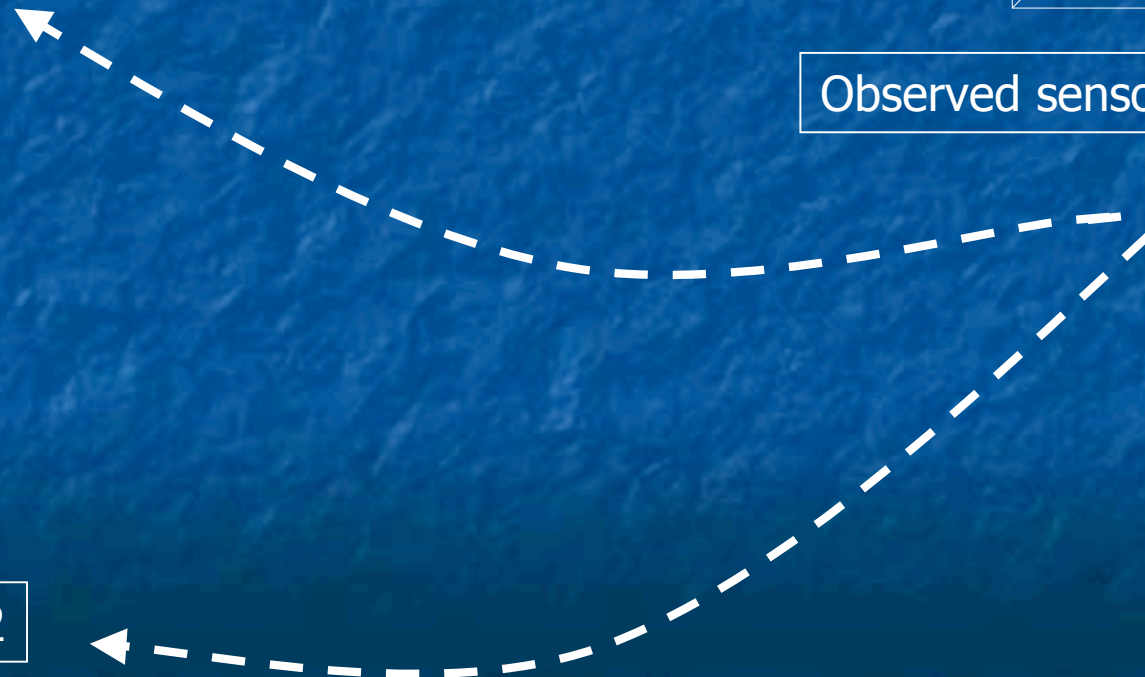
Perceived State S1



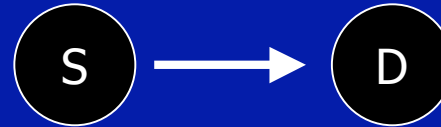
Observed sensory data D



Perceived State S2



A simplified perception scheme:



Sensory Data

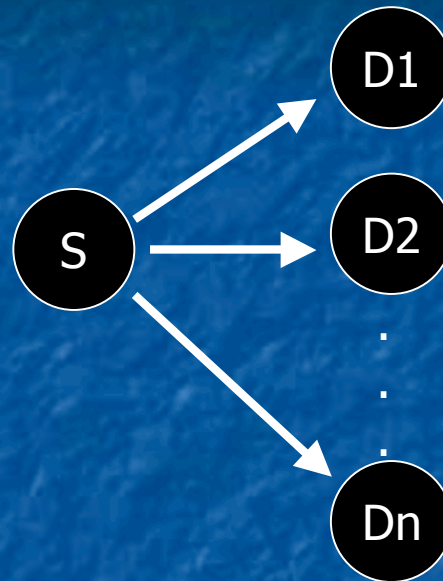
S : perceived state
 D : sensory data
Prior: $P(S)$
Sensor model: $P(D | S)$

Perception:
 $P(S | D) \sim P(S).P(D | S)$

Action



Perception viewed as an internal model explaining sensory data



$$P(S \mid D_1 D_2 \dots D_n) \sim P(S) \cdot \prod_{i=1,n} P(D_i \mid S)$$

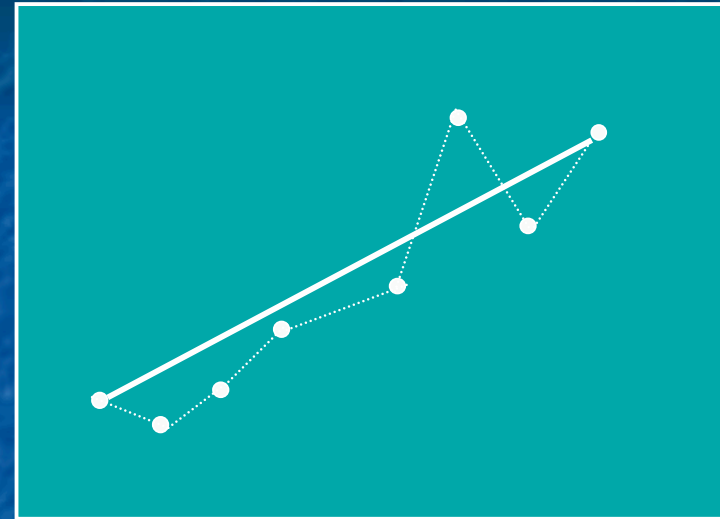
$$\log P(S \mid D_1 D_2 \dots D_n) = \alpha + \log P(S) + \sum_{i=1,n} \log P(D_i \mid S)$$

$$\text{Ex.: } \log P(S \mid D_1 D_2 \dots D_n) = \alpha - \frac{1}{2} |S|^2 - \frac{1}{2} \sum_{i=1,n} |D_i - S|^2 / \sigma_m^2$$

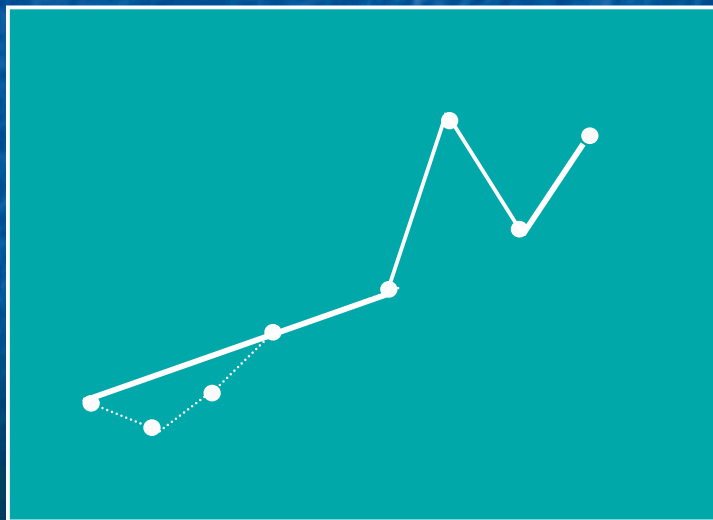
Ex. : polygonal segmentation from laser proximeter data $P(S) = (1/Z) \cdot \text{Exp}(-\beta \cdot \text{Nb of } 1)$
 β = regularization constant



S=11111111



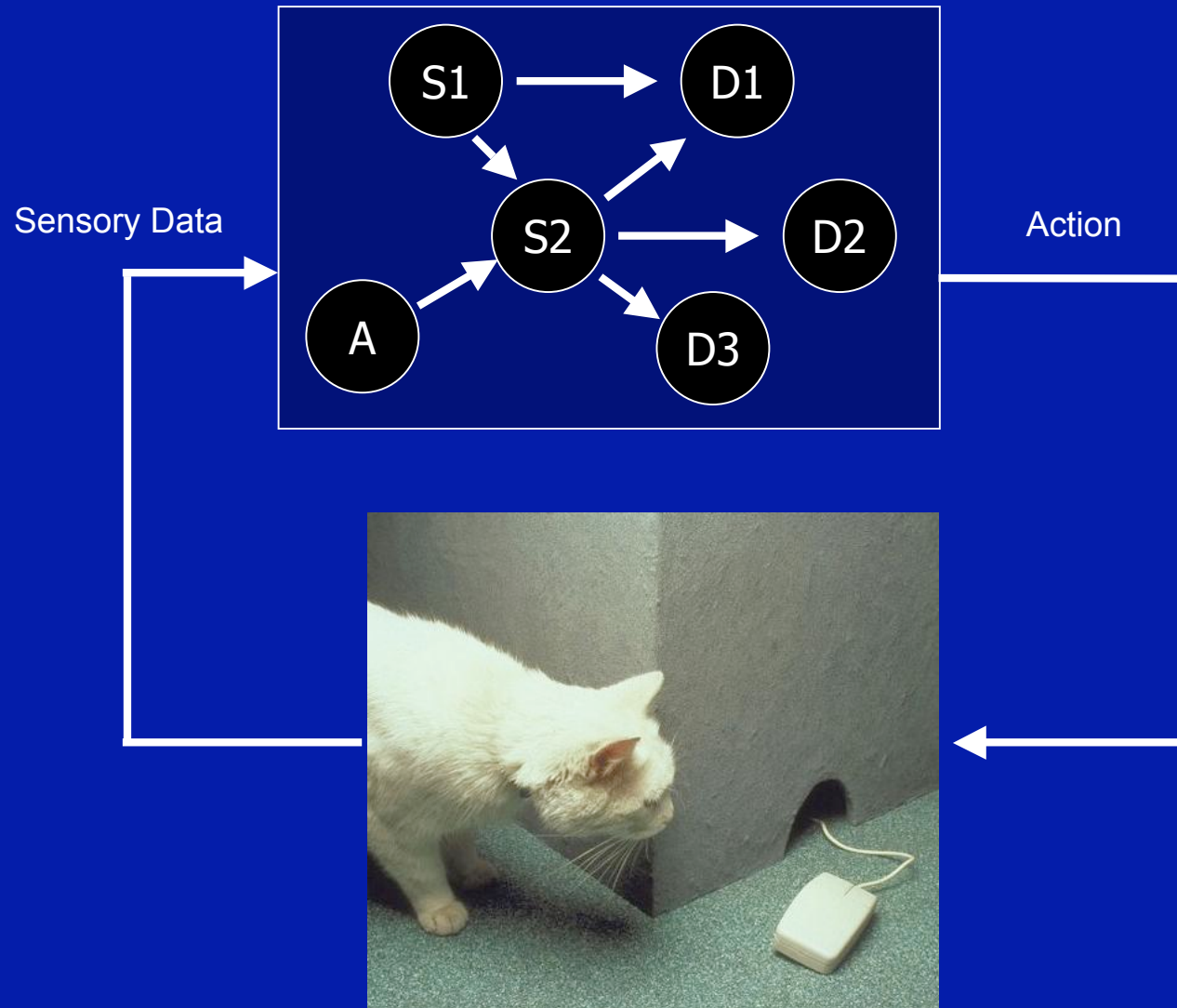
S=10000001



S=10001111



More complex structures can be captured by Bayesian networks

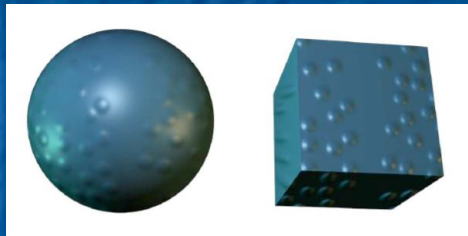


Two examples:

- 1. 3D shape from motion**
- 2. 3D self-motion perception**

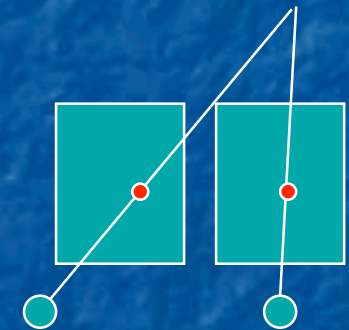
Object Perception

Numerous sources of information
Various characteristics (uncertainty, ambiguity)

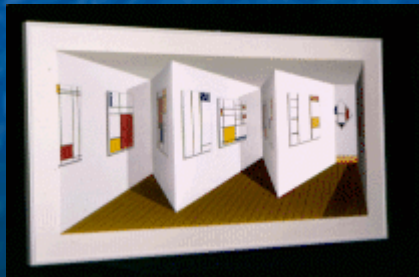


Shadows, reflexions

3D Structure-from-?



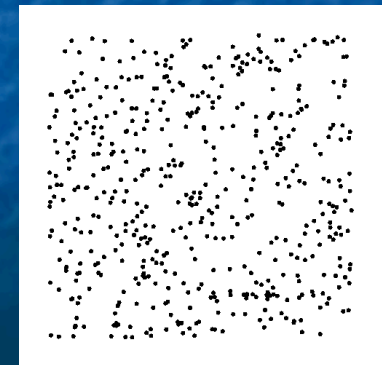
Binocular disparity



Perspective



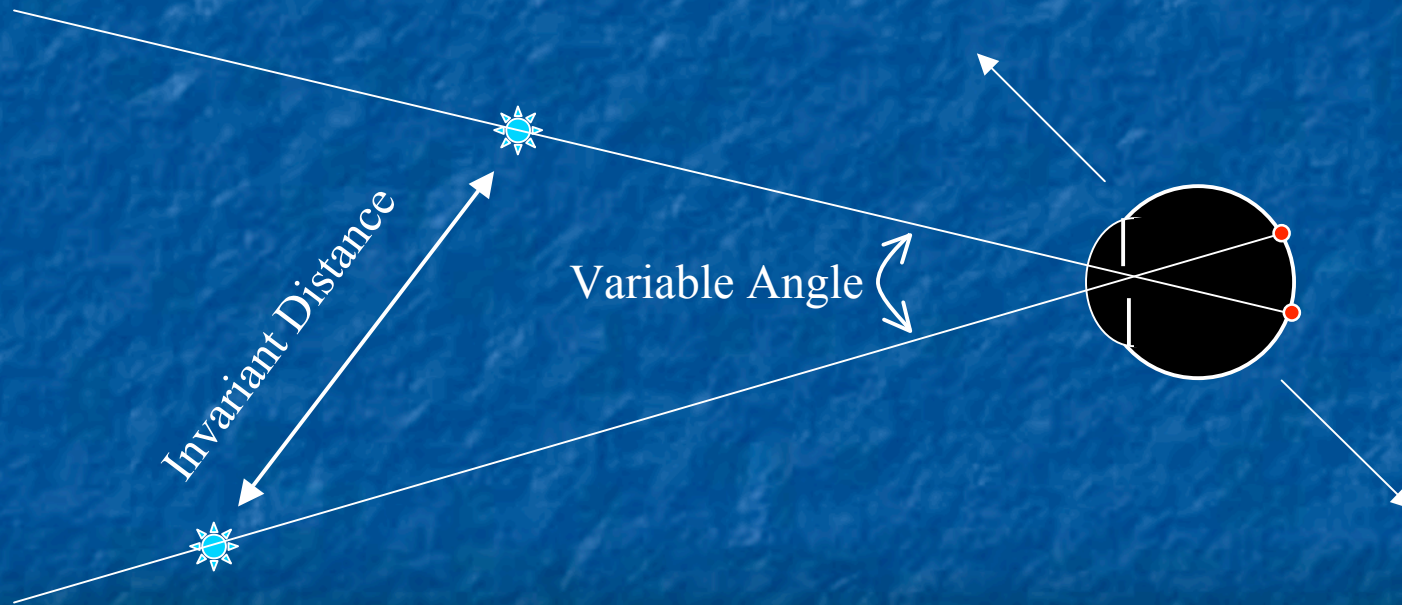
Colors & Textures



Movement

Motion cues:

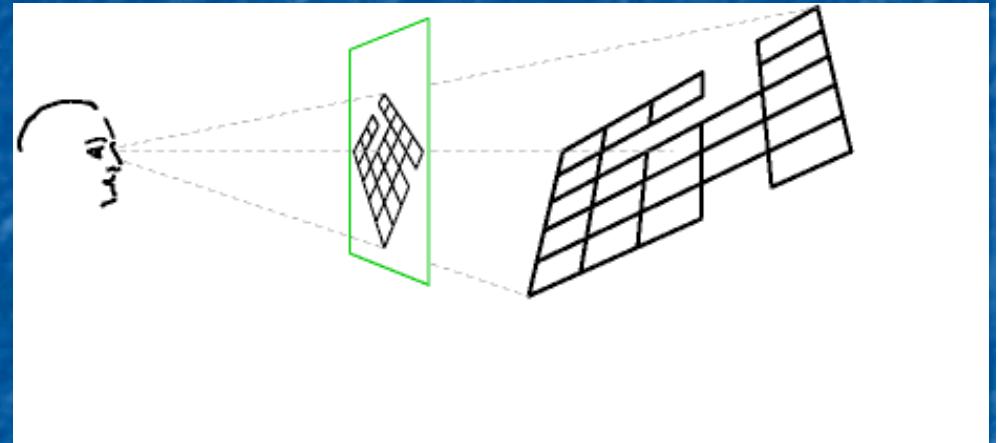
The rigidity assumption : the relative movement is a 3D isometric transformation



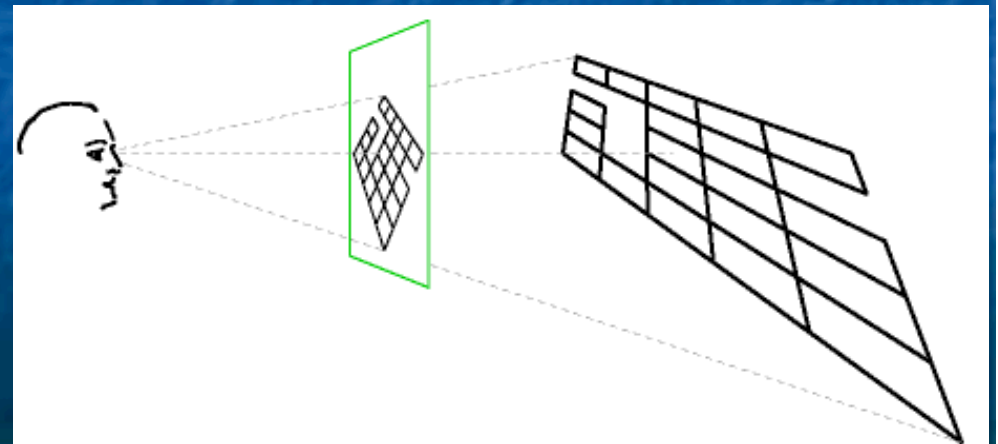
Perspective cues:

prior knowledge favoring regular texture on 3D surface

The image of a regular plane ...



...back projected on another plane
⇒ « trompe-l'œil »



According to the rigidity assumption:

Optic Flow = rotation + translation velocity fields

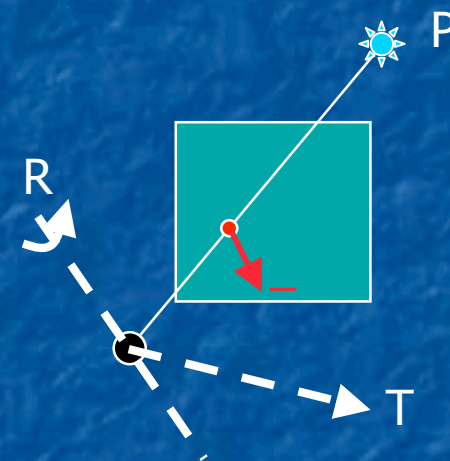
$$\underline{u} = R + p.T \quad p = \text{proximity map (1/distance)}$$

Object geometry (p) and relative 3D motion (R,T) determine the optic flow (\underline{u})
Knowing \underline{u} , how to compute shape and movement parameters (p, R, T) ?

The **direct functional model** is quite simple:

$$-u_x(x,y) = p(x,y).(T_x - x.T_z) + (1 + x^2).R_y - y.R_z - xy.R_x$$

$$-u_y(x,y) = p(x,y).(T_y - y.T_z) - (1 + y^2).R_x + x.R_z + xy.R_y$$

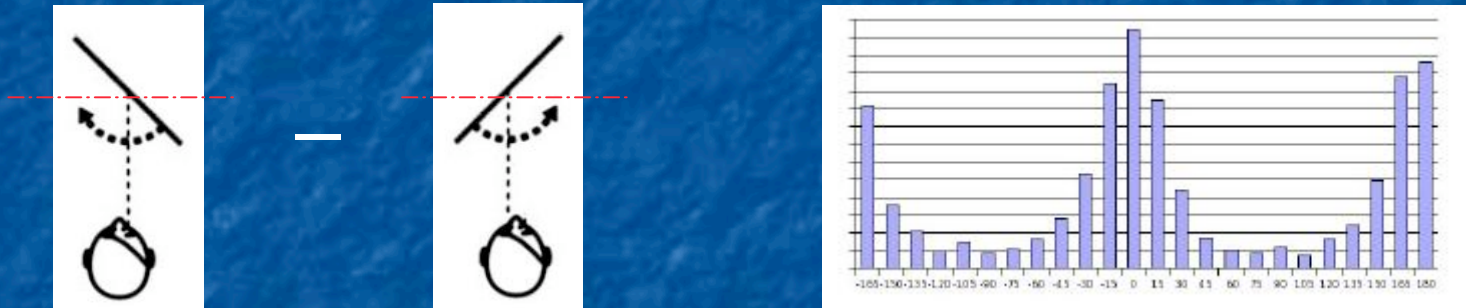


But the **inverse problem** is quite difficult ...

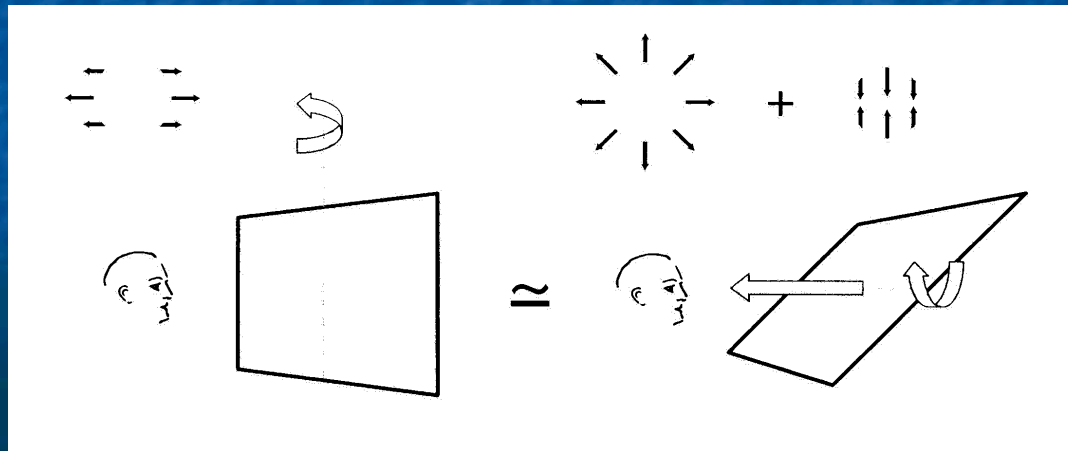
Non linear equations + “noise” + high dimension (~ 12)
 \Rightarrow General Algorithms are typically not robust

Several examples of optic flow ambiguities

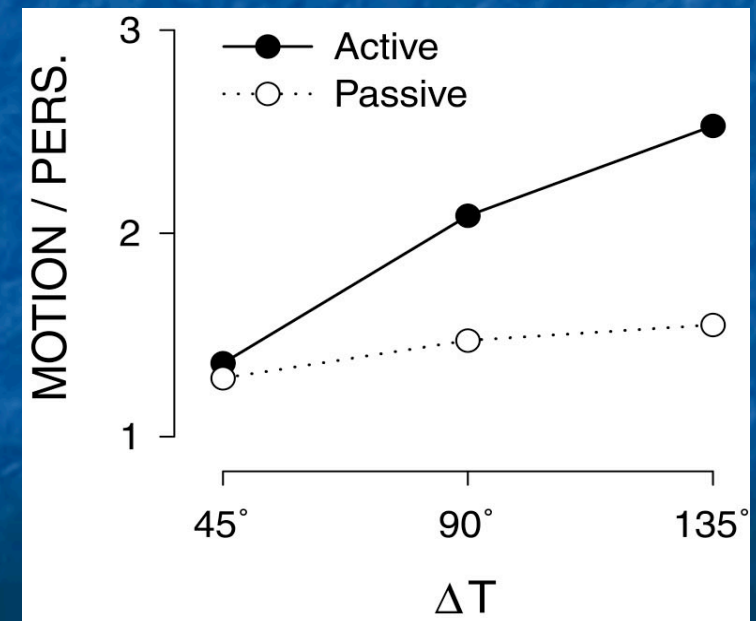
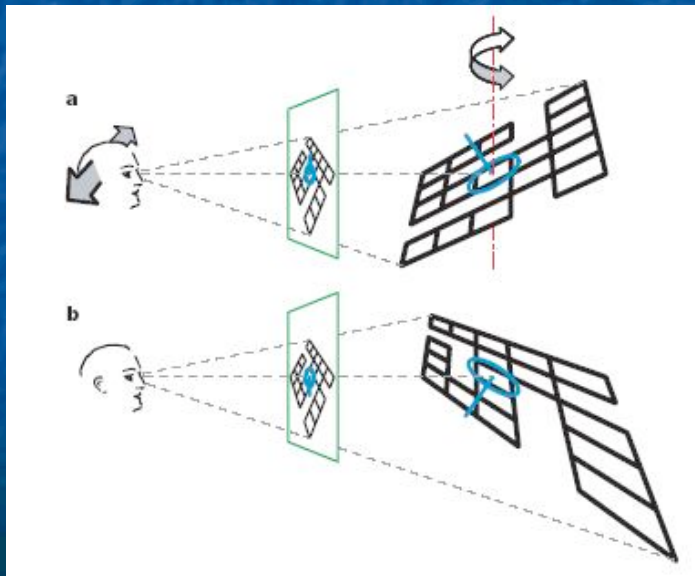
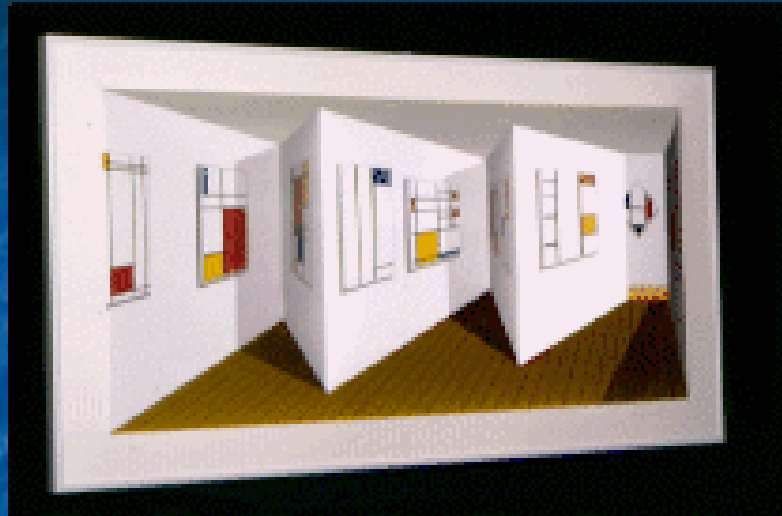
1. **Perceptive inversion** (Fronto-parallel plane symmetry for both object & motion)



2. Similar optic flows result from **different combinations of rotation and translation**



3. Conflict between motion & pictorial cues

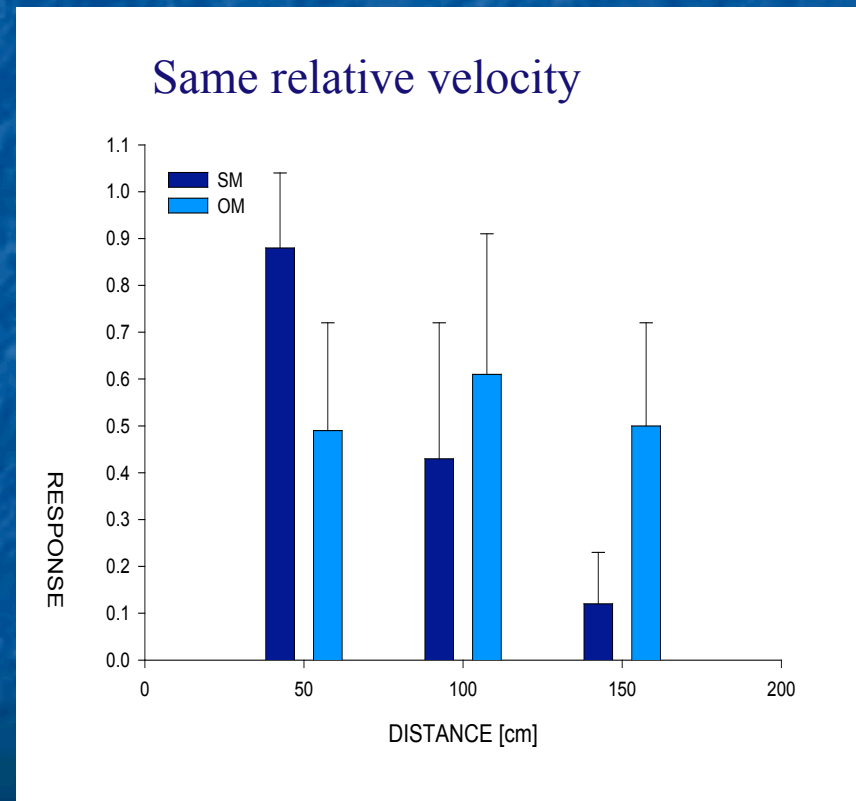
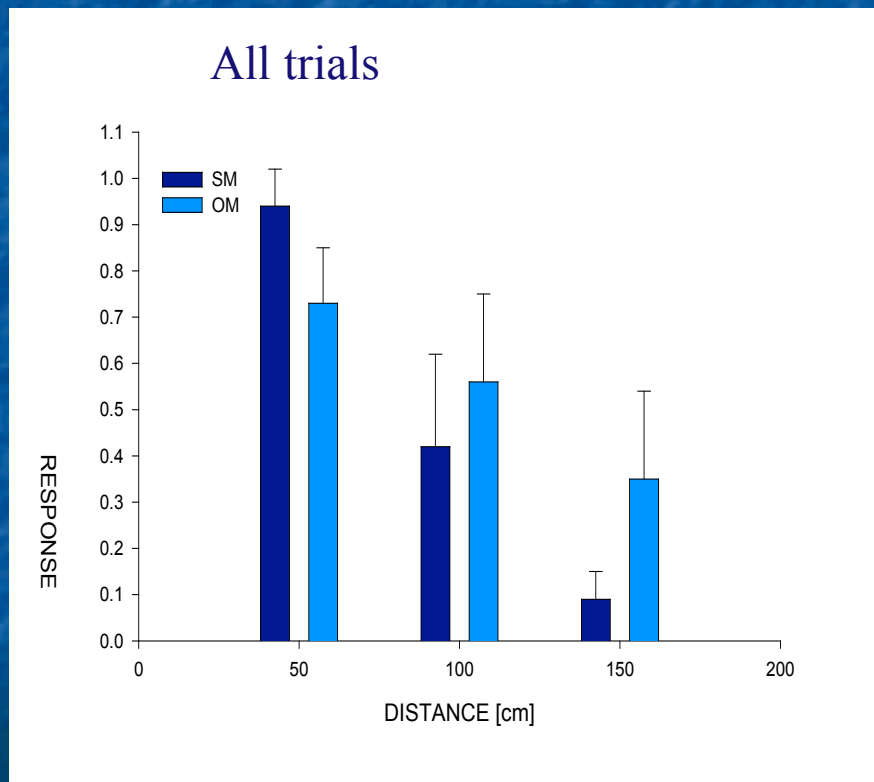


M. Wexler, F. Panerai, I. Lamouret & J. Droulez, Nature, 409, 85-88 (2001)

4. Contribution of self motion to depth perception (scale ambiguity)

Subject Motion (SM) versus Object Motion (OM)

Task: report whether or not object distance is less than one meter



Panerai, Cornilleau-Pérès & Droulez, Perception & Psychophysics, 64: 717-731 (2002)

The stationarity assumption :

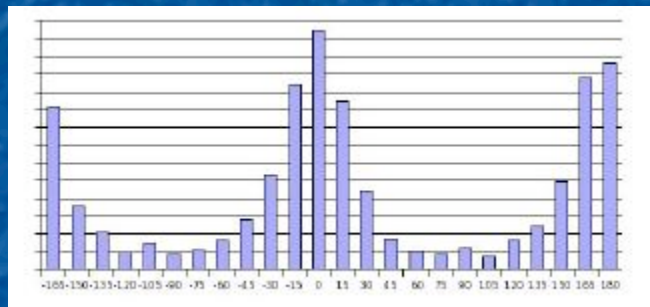
Preference for small allocentric object movement



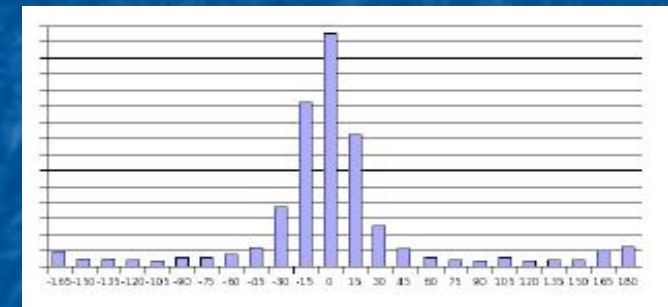
Knowledge about self motion (observer's displacement)

Can be used to remove optic flow ambiguities \Rightarrow **Stationarity Assumption**

Suppression of perceptive inversion



Same movement
but produced by
the observer



Preference for the most stationary solution (even if it is less rigid)

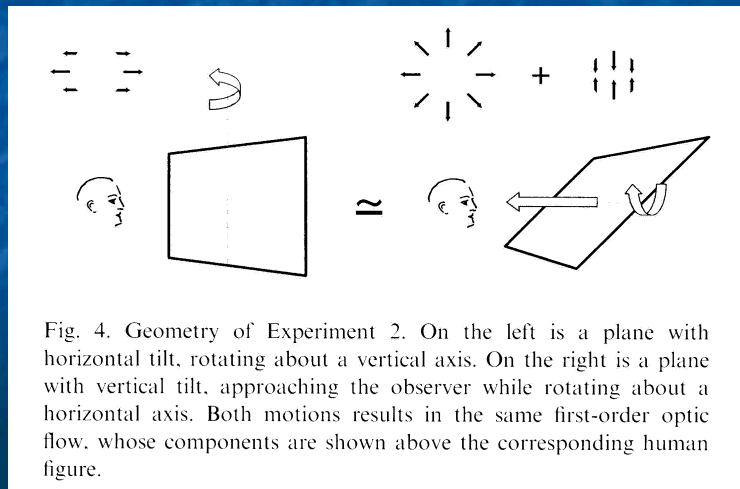


Fig. 4. Geometry of Experiment 2. On the left is a plane with horizontal tilt, rotating about a vertical axis. On the right is a plane with vertical tilt, approaching the observer while rotating about a horizontal axis. Both motions results in the same first-order optic flow, whose components are shown above the corresponding human figure.

Wexler, Lamouret & Droulez, Vision Research, 41, 3023-3037 (2001)

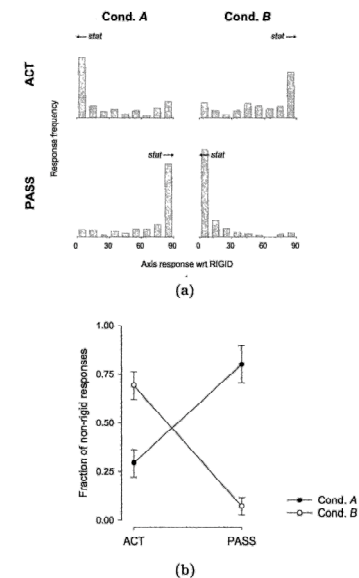
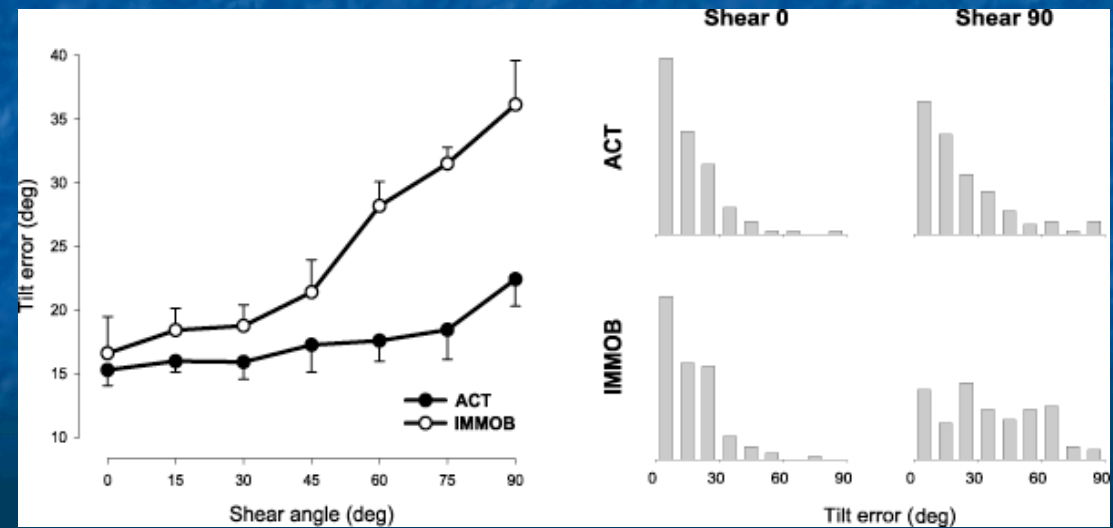
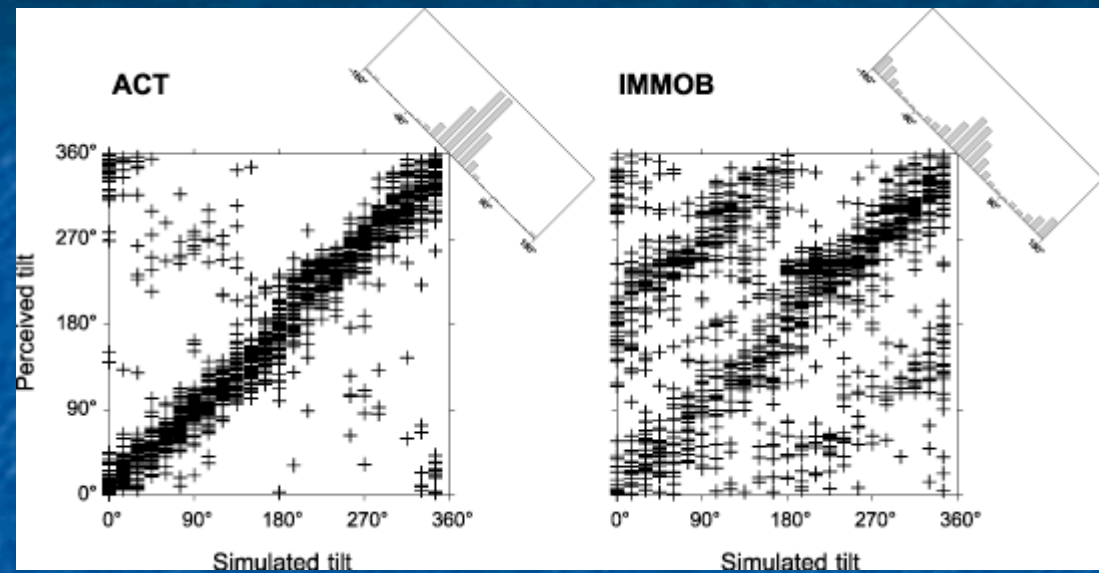
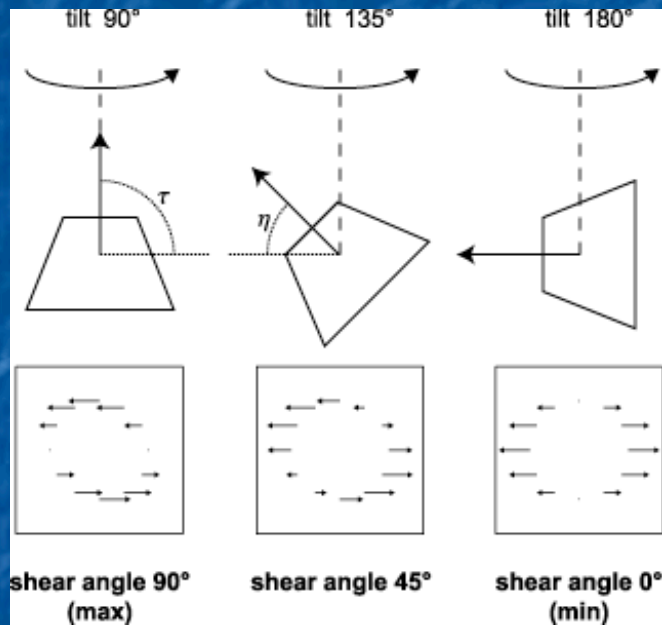


Fig. 6. Results of Experiment 2. (a) Distribution of axis responses for all subjects, in the ACT and PASS conditions, in conditions A and B. Response 0° indicates the rigid solution, 90° the deforming solution. Predictions of the stationary hypothesis are marked by 'stat'. (b) Fraction of responses closer to the non-rigid solution than to the rigid solution. Error bars represent between-subject standard errors.

Variability of perceptive responses (« shear effect »)



Structure of the probabilistic model

Variables : Object structure, Observer motion, Relative Motion, Optic Flow

Knowledge Expression:

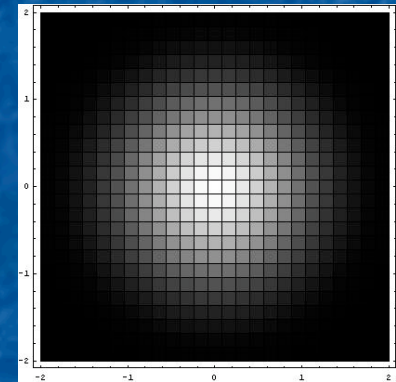
$$P(\text{Obj}, \text{Obs}, \text{Move}, \text{Flow}) = P(\text{Obj}) \cdot P(\text{Obs}) \cdot P(\text{Move} \mid \text{Obs}) \cdot P(\text{Flow} \mid \text{Move}, \text{Obj})$$

$P(\text{Obj})$ = “Fronto-parallel plane prior”

$P(\text{Obs})$ = “Self-motion information”

$P(\text{Move} \mid \text{Obs})$ = “Stationarity assumption”

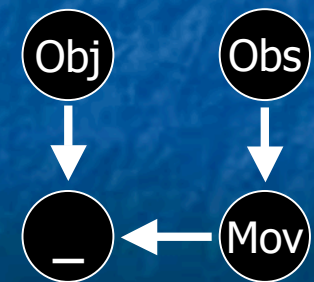
$P(\text{Flow} \mid \text{Move}, \text{Object})$ = “Rigidity assumption”



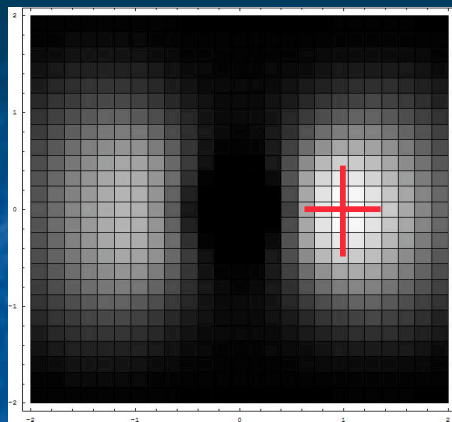
Question: $P(\text{Obj} \mid \text{Obs}, \text{Flow})$?

Experimental results to be explained:

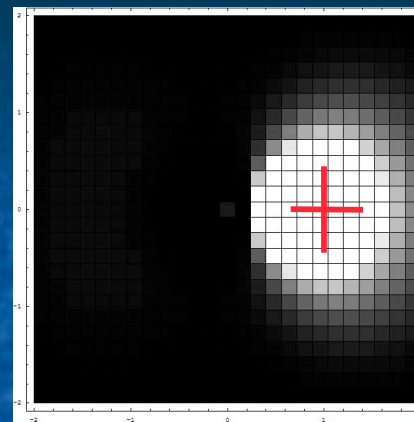
- Perceptive Inversion (suppressed in active condition)
- Perceptive variability due to shear (reduced in active condition)
- 90° Rotation of perceived orientation with added depth translation



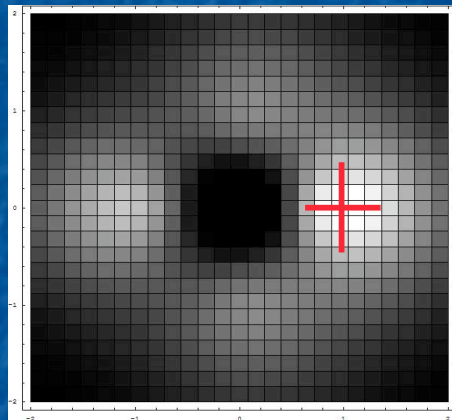
Probabilistic model (results)



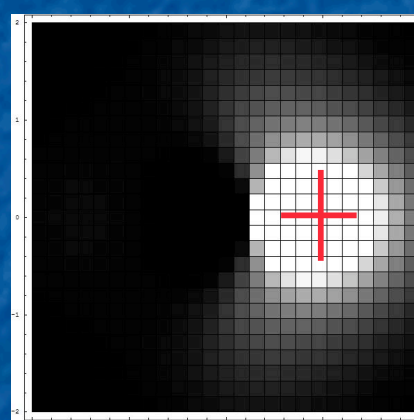
Immobile
Subject
Shear 0°



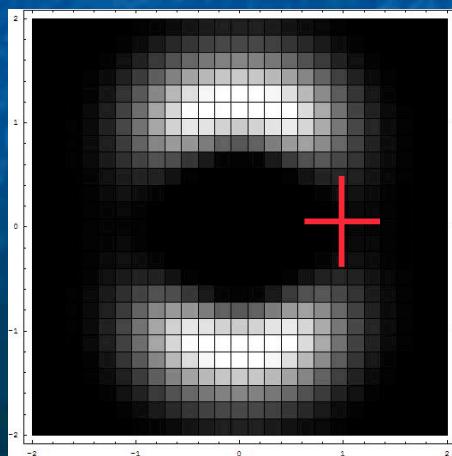
Moving
Subject
Shear 0°



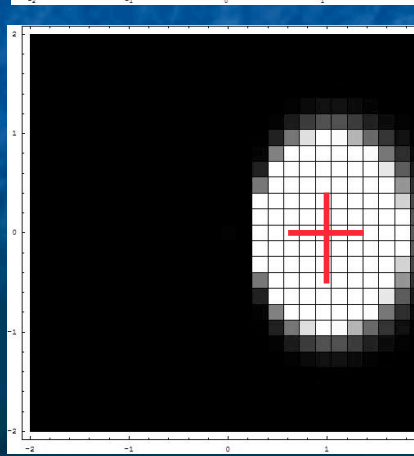
Immobile
Subject
Shear 90°



Moving
Subject
Shear 90°



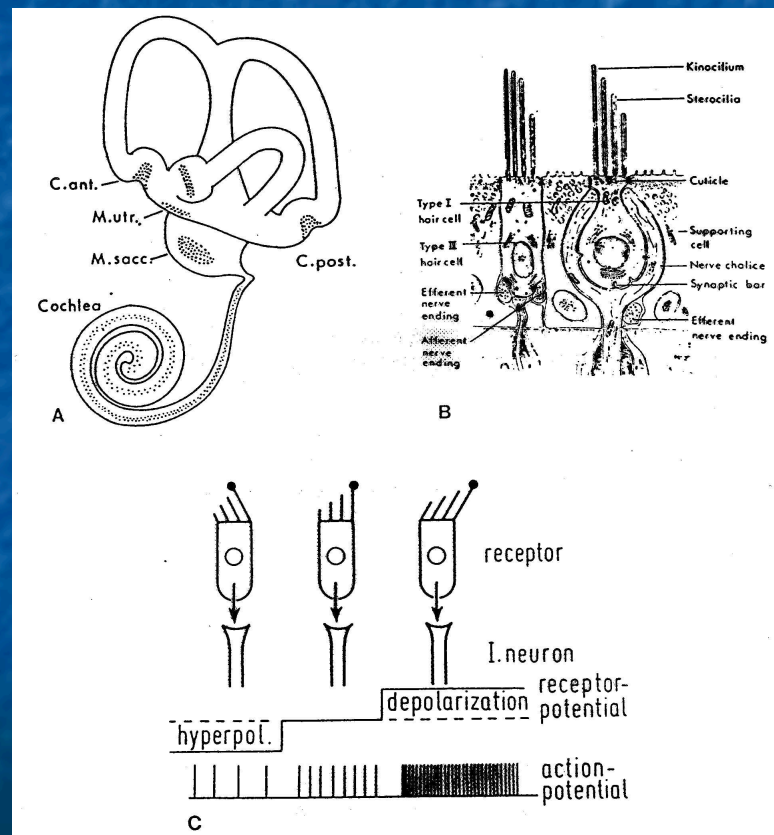
Immobile
Subject
+TZ



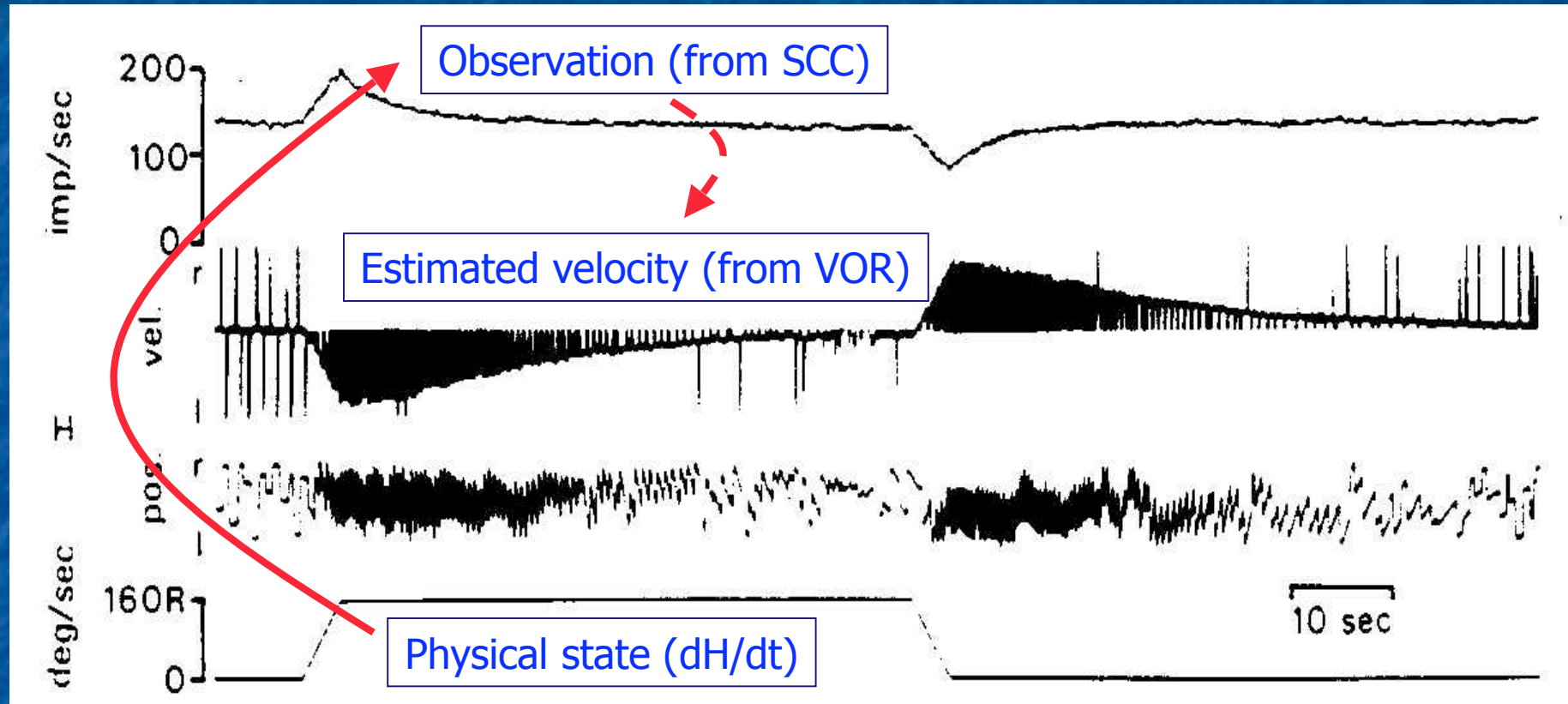
Moving
Subject
+TZ

2nd example: Self-motion perception

The vestibular sensor : 3 semi-circular canals (head angular acceleration)
+ 2 otolithic organs (head linear acceleration)



A first example of ambiguity: how to estimate the sustained angular velocity ?



Data from Büttner & Waespe (81)

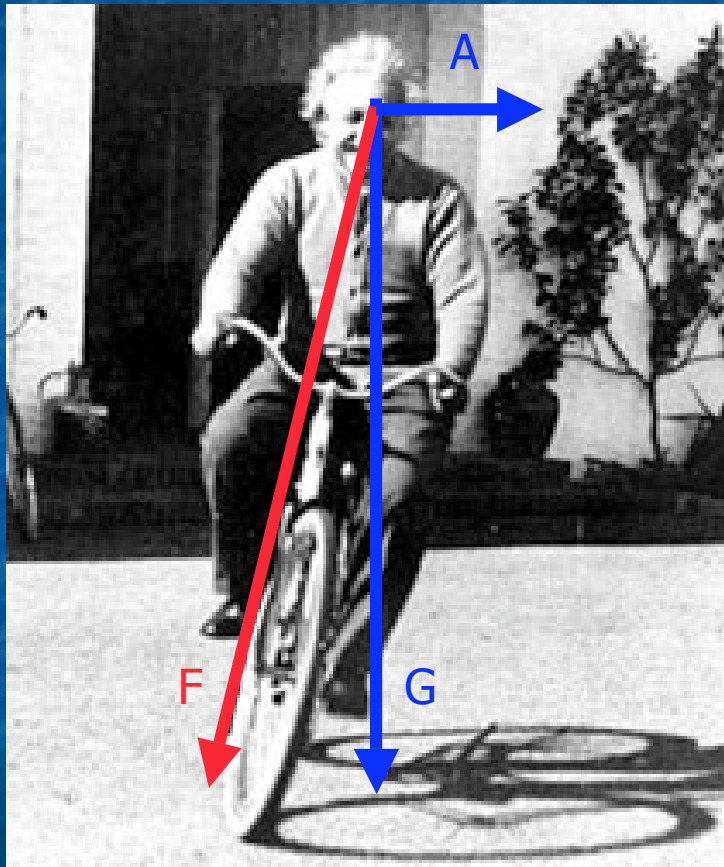
While an exact integration (from filtered acceleration to velocity) is mathematically straightforward, it would yield error accumulation with noisy sensory data !
⇒ The brain favors low estimated velocity

Another well-known example of ambiguity:
how to distinguish the inertial linear acceleration from gravity ?

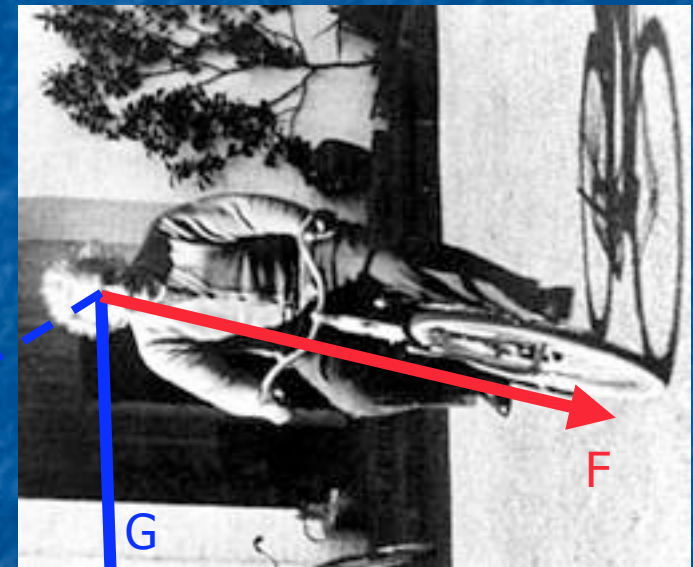


$$F = G - A$$

The physical state (A,G) cannot be inferred from the observed otolithic signal (F)



The actual solution

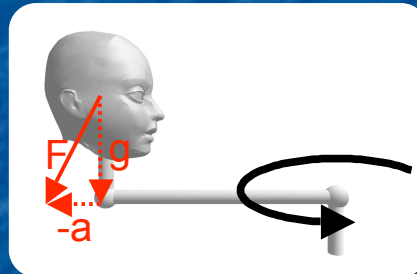


Another solution to the inverse problem

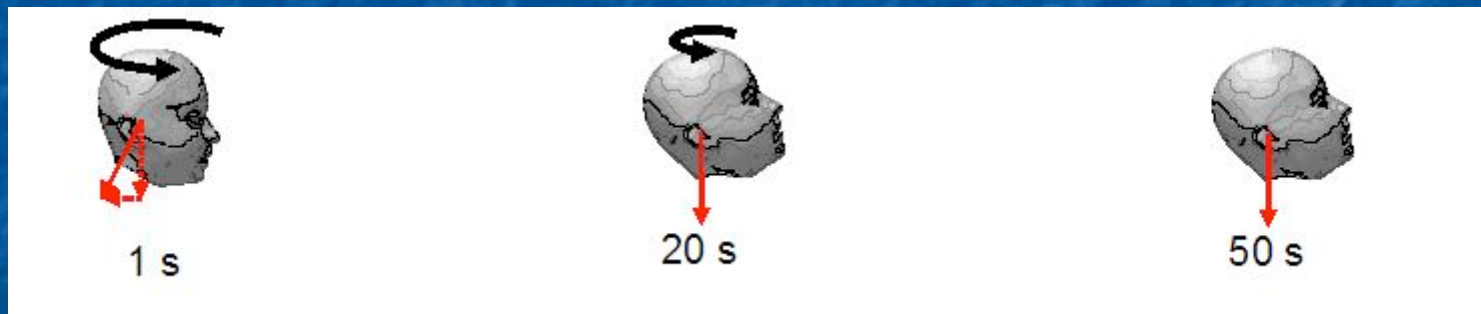
Both ambiguities combine each other !

Ex.: during off-axis rotation (centrifugation)

Physical state

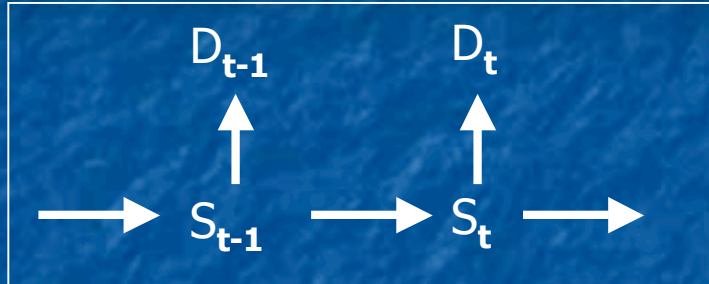


Perceived states



Decreasing the estimated angular velocity
⇒ alignment of estimated gravity with F
⇒ decreasing estimated linear acceleration to 0

Dealing with temporal series of variables: Bayesian Filters



Variables: $D_1, \dots, D_t, S_0, S_1, \dots, S_t$

Knowledge expression :

$$P(D_1, \dots, D_t, S_0, S_1, \dots, S_t) = P(S_0).P(S_1 | S_0).P(D_1 | S_1) \dots P(S_t | S_{t-1}).P(D_t | S_t)$$

Observation : $P(D_t | S_t)$ « sensor models »

Transition: $P(S_t | S_{t-1})$ « dynamic models »

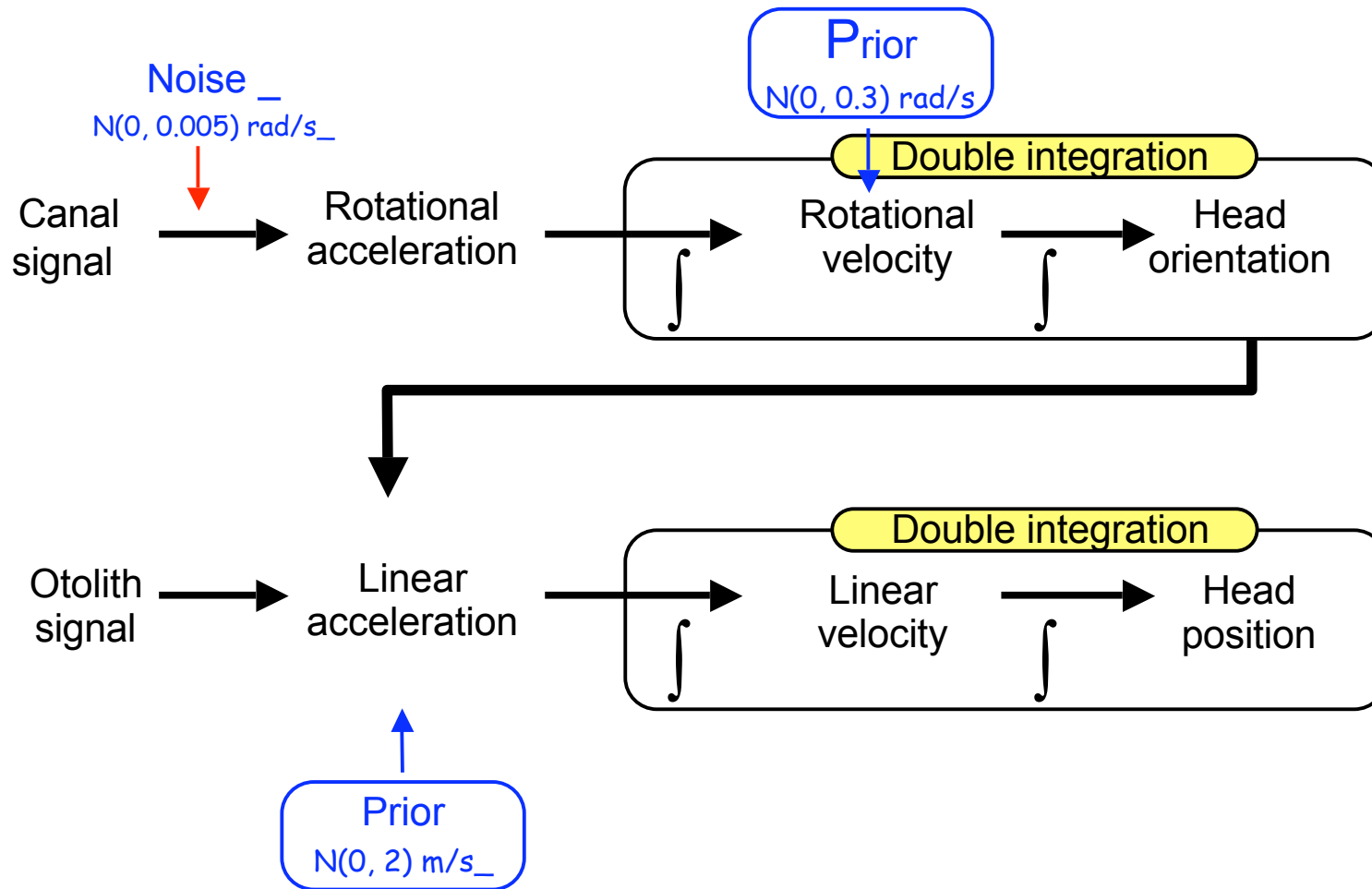
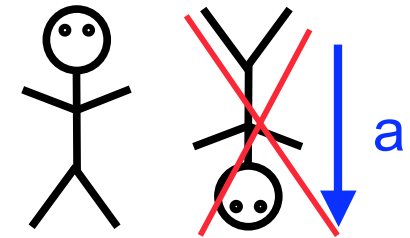
Exploitation:

$$P(S_t | d_1, \dots, d_t) \sim P(d_t | S_t) \cdot \sum_{S_{t-1}} P(S_t | S_{t-1}) \cdot P(S_{t-1} | d_1, \dots, d_{t-1})$$

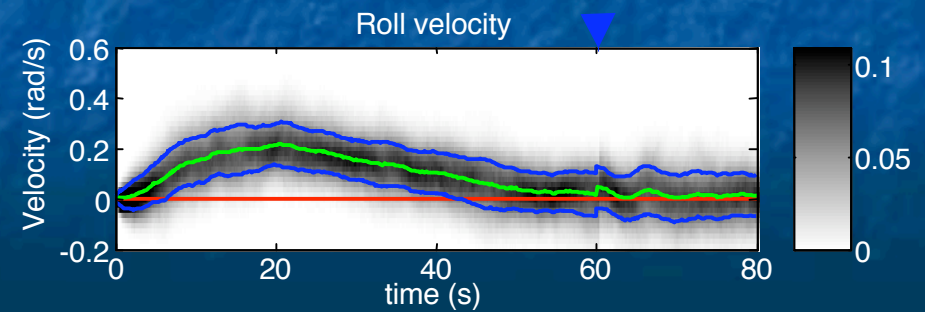
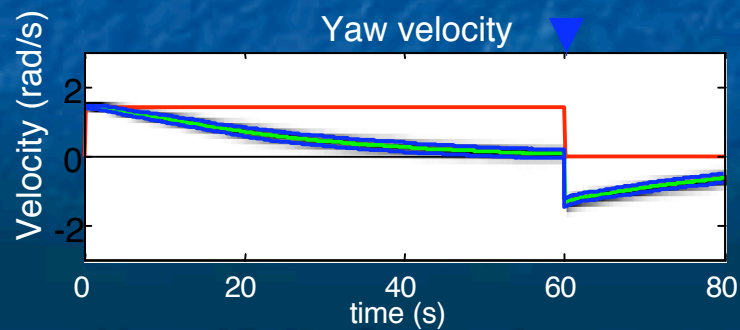
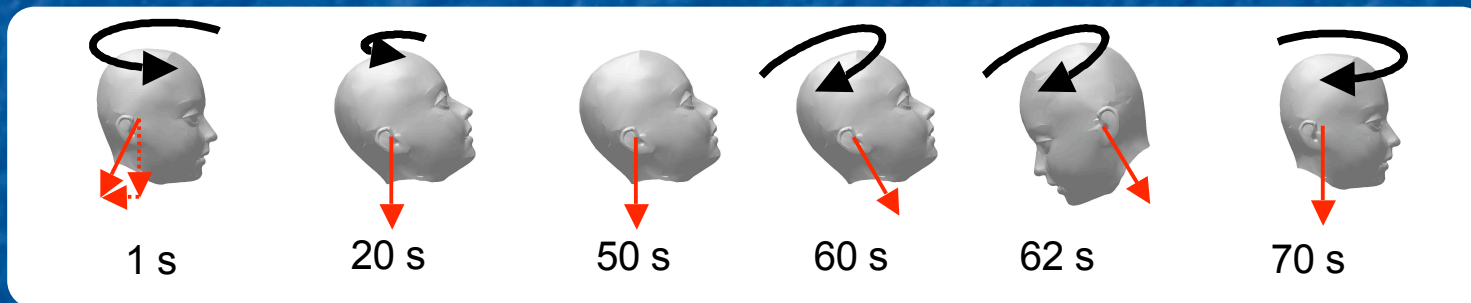
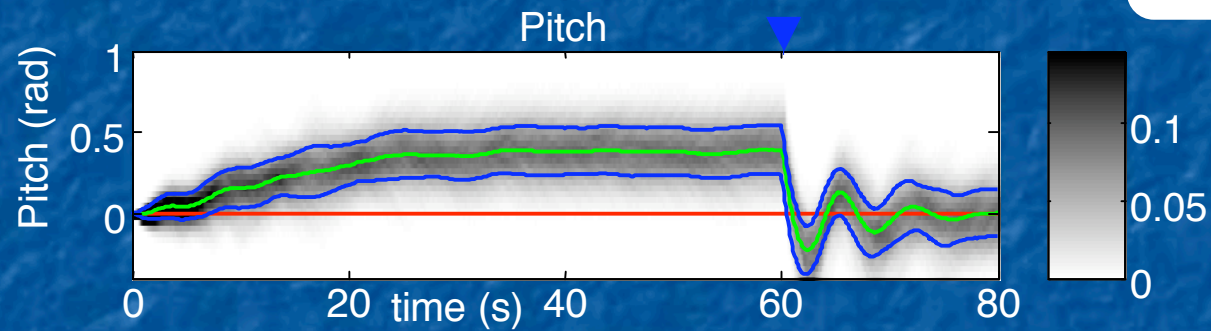
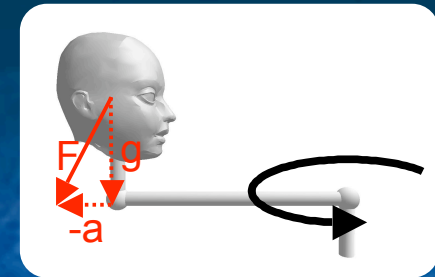
Particular cases: HMM, Kalman

Dynamics + priors

(Low angular velocity & linear acceleration)



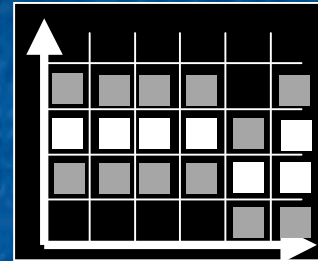
Centrifugation (Off Axis Rotation)



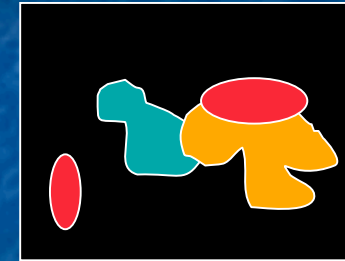
Neural Implementation of probabilistic computations: (the 3rd person viewpoint)

Main issues:

1. Relevant variables ?

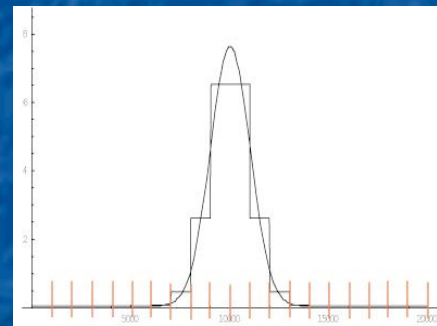


Coordinates: x_1, x_2, \dots

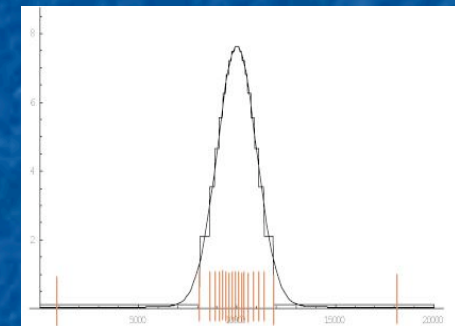


Space-time subsets: S_1, S_2, \dots

2. Neural code for $P(x)$?

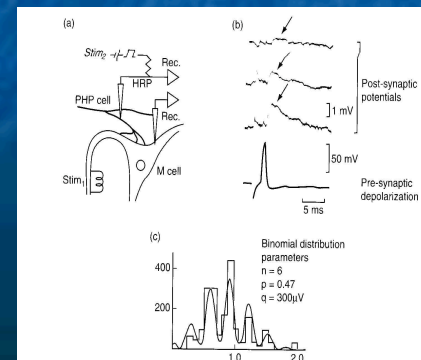


$$R_i(t) \sim \int_{\Delta t} p(x_{it}).dt$$



$$\int_{t_1}^{t_2} p(x_{it}).dt = \Delta$$

3. Reduction of computational costs ?



Sampling through
random neuroT
release

(H. Korn & DH Faber, 87)

Acknowledgements

Pierre Bessière
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Simon Capern
Francis Colas
Valérie Cornilleau-Pérès
Frédéric Davesne
Ivan Lamouret
Jean Laurens
Francesco Panerai
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Thank you for your attention,
and for your patience ...